

Topics covered in this tutorial

- Sums, convolution and product transformations of random variables
- Geometric probabilities over joint supports
- Gaussian-noise decision errors
- Marginal densities and conditioning on events

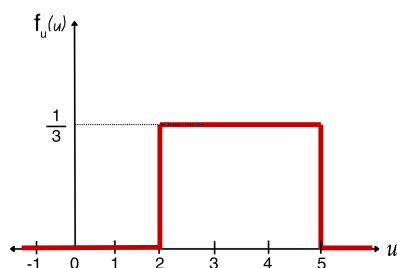
Complete all questions before the tutorial test that follows. The test normally starts between approximately 16:00 and 16:15.

Onderwerpe wat in hierdie tutoriaal gedek word

- Somme, konvolusie en produktransformasies van toevalsveranderlikes
- Meetkundige waarskynlikhede oor gesamentlike steungebiede
- Beslissingsfoute met Gaussiese ruis
- Marginale digthede en kondisionering op gebeurtenisse

Voltooi al die vrae voor die tutoriaaltoets wat daarop volg. Die toets begin gewoonlik tussen ongeveer 16:00 en 16:15.

1. Consider the independent random variables U and V with probability density functions $f_U(u)$ and $f_V(v)$ respectively.



- (a) Determine and sketch the PDF $f_Z(z)$ of $Z = U + V$.
(b) Use your result to calculate $P(Z > 6)$.

2. A player throws a dart towards a square board with side length 40 cm. Its landing position (U, V) is measured in centimetres from the board's centre $(0, 0)$. The bullseye is the disc of radius 3 cm at the centre. Calculate the probability $P(B)$ that the dart lands in the bullseye under each model below.

- (a) U and V are independent, identically distributed continuous uniform $(-20, 20)$ random variables.
(b) The joint PDF $f_{U,V}(u, v)$ is uniform over the disc of radius 20 cm centred at $(0, 0)$.
(c) U and V are independent, identically distributed Gaussian $(\mu = 0, \sigma = 8)$ random variables. In this model, a dart may land outside the board; such a throw is not a bullseye.

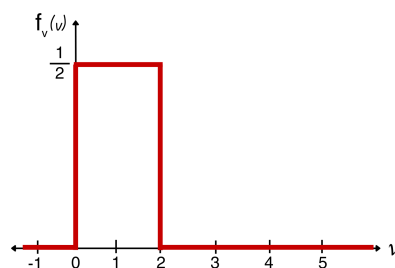
Hint for (c): Use polar coordinates.

$$\iint_R f(u, v) du dv = \iint_{R'} f(r \cos \theta, r \sin \theta) r dr d\theta.$$

3. First find the CDF and then the PDF of $W = UV$ when U and V have the joint PDF below. **Hint:** Sketch the support $S_{U,V}$ and the curve $uv = w$ for a fixed value of w in the u - v plane.

$$f_{U,V}(u, v) = \begin{cases} 1, & 0 \leq u \leq 1, \quad 0 \leq v \leq 2u, \\ 0, & \text{otherwise/andersins.} \end{cases}$$

- Beskou die onafhanklike toevalsveranderlikes U en V met waarskynlikheidsdigtheidsfunksies $f_U(u)$ en $f_V(v)$ onderskeidelik.



- (a) Bepaal en skets die WDF $f_Z(z)$ van $Z = U + V$.
(b) Gebruik jou antwoord om $P(Z > 6)$ te bereken.

- 'n Speler gooi 'n pyl na 'n vierkante bord met sy lengte 40 cm. Die landingsposisie (U, V) word in sentimeter vanaf die bord se middelpunt $(0, 0)$ gemeet. Die kol is die skyf met radius 3 cm by die middelpunt. Bereken die waarskynlikheid $P(B)$ dat die pyl in die kol land vir elke model hieronder.

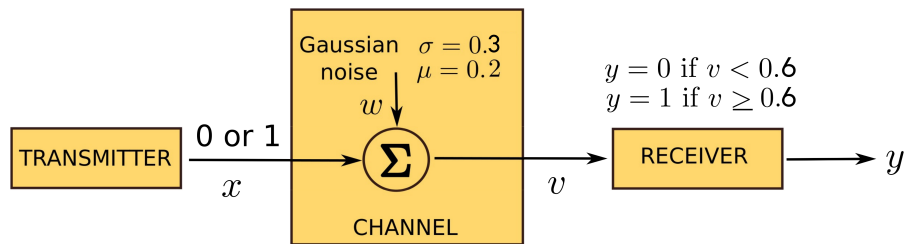
- (a) U en V is onafhanklike, identies verdeelde kontinue uniform $(-20, 20)$ toevalsveranderlikes.
(b) Die gesamentlike WDF $f_{U,V}(u, v)$ is uniform oor die skyf met radius 20 cm en middelpunt $(0, 0)$.
(c) U en V is onafhanklike, identies verdeelde Gaussiese $(\mu = 0, \sigma = 8)$ toevalsveranderlikes. In hierdie model kan 'n pyl buite die bord land; so 'n gooi tref nie die kol nie.

Wenk vir (c): Gebruik poolkoördinate.

- Vind eers die KVF en dan die WDF van $W = UV$ wanneer U en V die onderstaande gesamentlike WDF het. **Wenk:** Skets die steungebied $S_{U,V}$ en die kromme $uv = w$ vir 'n vaste waarde van w in die u - v -vlak.

4. The figure shows a communication system transmitting binary data.

Die figuur toon 'n kommunikasiestelsel wat binêre data versend.



The transmitter sends an amplitude of 0 or 1 with equal probability. Independent Gaussian noise, with mean 0.2 and standard deviation 0.3, is added to the transmitted amplitude. The receiver decodes a 1 if the received amplitude is at least 0.6, and a 0 otherwise.

Die sender stuur 'n amplitude van 0 of 1 met gelyke waarskynlikheid. Onafhanklike Gaussiese ruis, met gemiddeld 0.2 en standaardafwyking 0.3, word by die gestuurde amplitude getel. Die ontvanger dekodeer 'n 1 as die ontvangde amplitude minstens 0.6 is, en andersins 'n 0.

- (a) Given that a 1 was transmitted, what is the probability that it is decoded incorrectly?
(b) What is the probability that a randomly transmitted bit is decoded incorrectly?

- (a) Gegee dat 'n 1 gestuur is, wat is die waarskynlikheid dat dit verkeerd gedekodeer word?
(b) Wat is die waarskynlikheid dat 'n ewekansig gestuurde bis verkeerd gedekodeer word?

5. Random variables X and Y have joint PDF

Toevalsveranderlikes X en Y het gesamentlike WDF

$$f_{X,Y}(x,y) = \begin{cases} 1/2, & -1 \leq x \leq y \leq 1, \\ 0, & \text{otherwise/andersins.} \end{cases}$$

- (a) Determine the marginal PDF $f_Y(y)$.
(b) For the event $B = \{Y > 0\}$, calculate $P(B)$ and determine the complete conditional PDF $f_{X|B}(x)$.

- (a) Bepaal die marginale WDF $f_Y(y)$.
(b) Vir die gebeurtenis $B = \{Y > 0\}$, bereken $P(B)$ en bepaal die volledige voorwaardelike WDF $f_{X|B}(x)$.

6. Consider random variables U and V with the following joint CDF, where $b > 0$.

Beskou toevalsveranderlikes U en V met die volgende gesamentlike KVF, waar $b > 0$.

$$F_{U,V}(u,v) = \begin{cases} 1 - e^{-bu} - e^{-bv} + e^{-b(u+v)}, & u \geq 0, v \geq 0, \\ 0, & u < 0 \vee v < 0. \end{cases}$$

Let $B = \{V > 1\}$. Determine the complete conditional PDF $f_{U|B}(u)$ and explain whether learning that B occurred changes the distribution of U . Sketch the density for $b = 3$ over $-1 \leq u \leq 2$. Show your working clearly.

Laat $B = \{V > 1\}$. Bepaal die volledige voorwaardelike WDF $f_{U|B}(u)$ en verduidelik of die kennis dat B plaasgevind het die verdeling van U verander. Skets die digtheid vir $b = 3$ oor $-1 \leq u \leq 2$. Toon jou bewerkings duidelik.