Topics covered in this tutorial

- Continuous cumulative distribution and probability density functions
- Gaussian and exponential distributions
- Normalisation, probabilities and moments
- Mixed discrete–continuous random variables

Complete all questions in this tutorial before the tutorial test that follows. The test normally starts between approximately 16:00 and 16:15.

Onderwerpe wat in hierdie tutoriaal gedek word

- Kontinue kumulatiewe verdelings- en waarskynlikheidsdigtheidsfunksies
- Gaussiese en eksponensiële verdelings
- Normalisering, waarskynlikhede en momente
- Gemengde diskreet–kontinue toevalsveranderlikes

Voltooi al die vrae in hierdie tutoriaal voor die tutoriaaltoets wat daarop volg. Die toets begin gewoonlik tussen ongeveer 16:00 en 16:15.

---

1. Consider the following CDF for continuous random variable V

   Beskou die volgende KVF vir 'n kontinue toevalsveranderlike V

$$F_V(v) = \begin{cases} 0 & v < -5 \\ c(v+5)^2 & -5 \leq v < 7 \\ 1 & v \geq 7 \end{cases}$$

   Calculate:

   Bereken:

   (a) c
   (b) $P(V > 4)$
   (c) $P(-3 < V \leq 0)$
   (d) $P(|V| \leq 4)$
   (e) The value of a such that $P(V > a) = \frac{2}{3}$.
   Die waarde van a sodat $P(V > a) = \frac{2}{3}$.

2. A Gaussian random variable X has an expected value $\mathrm{E}[X] = 8$, and a variance $\sigma_x^2 = 4$. Determine the following:

   'n Gaussiese toevalsveranderlike X het 'n verwagte waarde $\mathrm{E}[X] = 8$, en 'n variansie $\sigma_X^2 = 4$. Bepaal die volgende:

   (a) $P(X \leq 5)$
   (b) $P(4 < X \leq 6)$
   (c) $P(X > 9)$

3. The midday wind speed W, in km/h, at a coastal town is a Gaussian random variable with a variance of 64. The wind speed W exceeds 20 km/h with probability 0.5. What is the probability that on a random day the wind speed is above 30 km/h?

   Die middagwindspoed W, in km/h, by 'n kusdorp is 'n Gaussiese toevalsveranderlike met 'n variansie van 64. Die windspoed W gaan 20 km/h verby met 'n waarskynlikheid van 0.5. Wat is die waarskynlikheid dat die windspoed op 'n ewekansige dag bo 30 km/h is?

4. Your cellphone battery is faulty and drains quickly. The amount of time X (in hours) that the phone lasts after being fully charged follows an exponential distribution with the CDF given below. Each charge cycle is independent.

Jou selfoon se battery is foutief en dreineer vinnig. Die hoeveelheid tyd X (in ure) wat die foon hou nadat dit heeltemal gelaai is volg 'n eksponensiele verdeling met die CDF wat hieronder gegee word. Elke laaisiklus is onafhanklik.

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-3x} & 0 \leq x \end{cases}$$

(a) What is the mean number of times per hour that the cellphone battery dies?

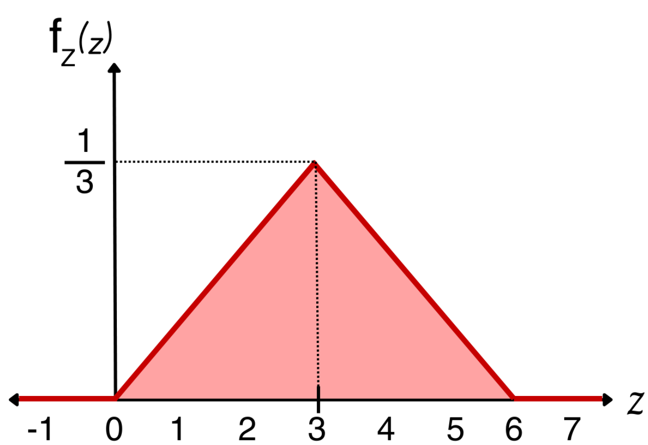
Wat is die gemiddelde aantal kere per uur wat die selfoonbattery doodgaan?

(b) You need to attend a 15-minute online meeting. Suppose the phone has just died, has then been freshly fully charged, and you take it off the charger at that moment so that the waiting time starts immediately. What is the probability that the phone will last through the full 15 minutes of the meeting without switching off?

Jy moet 'n 15-minute aanlyn vergadering bywoon. Veronderstel die foon het net doodgegaan, is daarna vars heeltemal gelaai, en jy haal dit op daardie oomblik van die laaier af sodat die wagtyd onmiddellik begin. Wat is die waarskynlikheid dat die foon die volle 15 minute van die vergadering kan uithou sonder om af te skakel?

5. A random variable Z has the probability density function shown below.

'n Toevalverandelike Z het die waarskynlikheidsdigtheidsfunksie soos onder getoon.

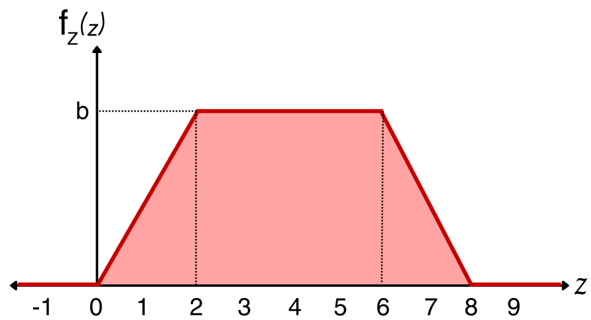


The first moment about the origin for this $f_Z(z)$ is $m_1 = 3$. Calculate the second central moment σ_Z^2 (i.e. the variance). Show and motivate your working clearly.

Die eerste moment om die oorsprong vir hierdie $f_Z(z)$ is $m_1 = 3$. Bereken die tweede sentrale moment σ_Z^2 (d.w.s. die variansie). Wys en motiveer u bewerkings duidelik.

6. The probability density function describing the delivery time (in hours) for a courier service is shown below.

Die waarskynlikheidsdigtheidsfunksie onder beskryf die afleweringstyd (in ure) by 'n koerierdiens.



(a) Compute b.

Bereken b.

(b) A customer needs their package delivered within 5 hours. What is the probability that the delivery will be completed on time?

'n Klient benodig hul pakkie binne 5 uur afgelewer. Bereken die waarskynlikheid dat die aflewering betyds voltooi sal word.

(c) Sketch the cumulative probability distribution function $F_Z(z)$.

Skets die kumulatiewe waarskynlikheisverdeling $F_Z(z)$.

7. A temperature sensor sometimes fails. With probability 0.05, it outputs exactly -50 (an error code). Otherwise, the reading Z is uniformly distributed between 10°C and 30°C.

'n Temperatuursensor faal soms. Met waarskynlikheid 0.05 gee dit presies -50 ('n foutkode). Andersins is die lesing Z eweredig versprei tussen 10°C en 30°C.

(a)	Write down the mixed PDF of Z.	Skryf die gemengde PDF van Z neer.
(b)	Find the CDF $F_Z(z)$.	Bepaal die CDF $F_Z(z)$.
(c)	What is the probability that the sensor reading indicates an error?	Wat is die waarskynlikheid dat die sensorlesing 'n fout aandui?
(d)	Compute the probability that the reading is above 25°C.	Bereken die waarskynlikheid dat die lesing bo 25°C is.
(e)	Compute the expected sensor reading $E[Z]$.	Bereken die verwagte sensorlesing $E[Z]$.