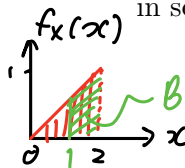


Tutorial Test 7 - Tutoriaal Toets 7 (20 minutes)

Name and surname / Naam en van	Memo	Mark / Punt [10]
Student number / Studentenommer		
Mini formula sheet / Mini-formuleblad		
$f_{X B}(x) = \begin{cases} \frac{f_X(x)}{P(B)}, & x \in B, \\ 0, & x \notin B. \end{cases}$		$W = g(X)$ $F_W(w) = P(g(X) \leq w)$

A data packet has delivery delay X , measured in seconds, with PDF

$$f_X(x) = \begin{cases} x/2, & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$



'n Datapakkie se afleweringvertraging X , gemeet in sekondes, het WDF

$$f_X(x) = \begin{cases} x/2, & 0 < x < 2, \\ 0, & \text{andersins.} \end{cases}$$

A packet is flagged as late if its delay exceeds one second. Let $B = \{X > 1\}$ denote this event.

'n Pakkie word as laat gemerk indien sy vertraging een sekonde oorskry. Laat $B = \{X > 1\}$ hierdie gebeurtenis aandui.

- Show that $P(B) = 3/4$. [2]
- Give the complete piecewise conditional PDF $f_{X|B}(x)$ of the delay of a packet flagged as late. [2]
- For a packet flagged as late, calculate the probability that its delay exceeds 1.5 seconds, $P(X > 3/2 | B)$. [2]
- The monitor assigns the delay score $W = X^2$, giving greater weight to long delays. Derive the complete piecewise CDF $F_W(w)$ for all packets, not only those flagged as late. Show your working. [4]

- Toon aan dat $P(B) = 3/4$. [2]
- Gee die volledige stuksgewyse voorwaardelike WDF $f_{X|B}(x)$ van die vertraging van 'n pakkie wat as laat gemerk is. [2]
- Bereken vir 'n pakkie wat as laat gemerk is die waarskynlikheid dat sy vertraging 1.5 sekondes oorskry, $P(X > 3/2 | B)$. [2]
- Die monitor ken die vertragingstelling $W = X^2$ toe, wat groter gewig aan lang vertraging gee. Lei die volledige stuksgewyse KVF $F_W(w)$ af vir alle pakkies, nie slegs dié wat as laat gemerk is nie. Toon jou berekening. [4]

$$a) P(B) = \int_1^2 \frac{x}{2} dx = \frac{1}{4} x^2 \Big|_1^2 = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}$$

✓ B def.

$$\left(\text{or } P(B) = 1 - \int_0^1 \frac{x}{2} dx = 1 - \frac{1}{2} \cdot 1 \cdot 0.5 = \frac{3}{4} \right)$$

✓ Process

$$b) f_{X|B}(x) = \begin{cases} \frac{f_X(x)}{P(B)} & x \in B \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{x/2}{3/4} = \frac{2}{3}x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

✓ interval

✓ otherwise

$$\begin{aligned}
 c) P(X > 1,5 | B) &= \int_{1,5}^2 f_{X|B}(x) dx \quad \checkmark \text{ setup} \quad (1,5 = \frac{3}{2}) \\
 &= \int_{1,5}^2 \frac{2}{3} x dx \\
 &= \frac{1}{3} x^2 \Big|_{1,5}^2 = \frac{4}{3} - \left(\frac{9}{4}\right) \cdot \frac{1}{3} \\
 &= \frac{7}{12} = 0,58\bar{3} \quad \checkmark \text{ answer.}
 \end{aligned}$$

d) for $w = x^2$ with $0 < x < 2 \rightarrow 0 < w < 4$

$$F_w(w) = P(W \leq w) = P(X^2 \leq w) \quad \checkmark = P(-\sqrt{w} \leq x \leq \sqrt{w})$$

$$\text{but since } x > 0 = P(0 \leq x \leq \sqrt{w}) \quad \checkmark$$

$$= F_x(\sqrt{w})$$

$$= \int_0^{\sqrt{w}} f_x(x) dx$$

$$= \int_0^{\sqrt{w}} \frac{x}{2} dx$$

$$= \frac{1}{4} x^2 \Big|_0^{\sqrt{w}}$$

$$= \begin{cases} 0 & w \leq 0 \quad \checkmark \\ \frac{w}{4} & 0 < w < 4 \quad \checkmark \\ 1 & w > 4 \quad \checkmark \end{cases}$$

or alternatively: $w = g(x) = x^2$ is monotonically increasing.

$$\therefore f_w(w) = \frac{f_x(x)}{\left| \frac{\partial g(x)}{\partial x} \right|_{x=g^{-1}(w)}} \quad \checkmark \quad g^{-1}(w) = \sqrt{w} \quad \frac{\partial g(x)}{\partial x} = 2x$$

$$= \frac{\frac{x}{2}}{2x} \Big|_{x=\sqrt{w}} = \begin{cases} \frac{1}{4} & 0 < w < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$F_W(w) = \int_0^w \frac{1}{4} du = \frac{1}{4} u \Big|_0^w = \frac{w}{4}$$

$$= \begin{cases} 0 & w \leq 0 \\ \frac{w}{4} & 0 < w < 4 \\ 1 & w \geq 4 \end{cases}$$