

Tutorial Test 6 - Tutoriaal Toets 6 (²⁰~~15~~ minutes)

Name and surname / Naam en van	Memo	Mark / Punt [12]
Student number / Studentenommer		12

Mini formula sheet / Mini-formuleblad

$E[g(X)] = \sum_{x \in S_X} g(x)P_X(x)$	$P_X(x) = \sum_y P_{X,Y}(x, y)$	$\text{Cov}[X, Y] = E[XY] - E[X]E[Y]$
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Question / Vraag 1 [12]

Discrete random variables X and Y , with respective supports $S_X = \{0, 1\}$ and $S_Y = \{-1, 1, 2\}$, have joint PMF

$P_{X,Y}(x, y)$	$y = -1$	$y = 1$	$y = 2$
$x = 0$	0.10	0.15	0.05
$x = 1$	0.20	a	0.10

Die diskrete toevalsveranderlikes X en Y , met onderskeie steunversamelings $S_X = \{0, 1\}$ en $S_Y = \{-1, 1, 2\}$, het die gegewe gesamentlike WMF.

$P_{X,Y}(x, y)$	$y = -1$	$y = 1$	$y = 2$
$x = 0$	0.10	0.15	0.05
$x = 1$	0.20	a	0.10

- a) Determine a . [2]
 b) Determine the marginal PMFs $P_X(x)$ and $P_Y(y)$. [3]
 c) Are X and Y independent? Show one decisive comparison. [2]
 d) Calculate $\text{Cov}[X, Y]$. [5]

- a) Bepaal a . [2] **2**
 b) Bepaal die marginale WMF's $P_X(x)$ en $P_Y(y)$. [3] **3**
 c) Is X en Y onafhanklik? Toon een beslissende vergelyking. [2] **2**
 d) Bereken $\text{Cov}[X, Y]$. [5] **5**

$$a) \sum_{y \in S_Y} \sum_{x \in S_X} P_{X,Y}(x, y) = 1$$

$$0,1 + 0,15 + 0,05 + 0,2 + a + 0,1 = 1$$

$$a = 0,4$$

$$b) P_X(x) = \begin{cases} 0,1 + 0,15 + 0,05 = 0,3 & x = 0 \\ 0,2 + 0,4 + 0,1 = 0,7 & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P_Y(y) = \begin{cases} 0,1 + 0,2 = 0,3 & y = -1 \\ 0,15 + 0,4 = 0,55 & y = 1 \\ 0,05 + 0,1 = 0,15 & y = 2 \\ 0 & \text{otherwise} \end{cases}$$

✓ full distribution (0 otherwise)

c) For independence $P_{X,Y}(x,y) = P_X(x) \cdot P_Y(y)$ ✓

$$P_{X,Y}(0,-1) = 0,1 \neq P_X(0) \cdot P_Y(-1) = 0,3 \cdot 0,3 = 0,09$$

∴ Not Independent ✓ (with reason)

d) $\text{Cov}[X,Y] = E[XY] - E[X]E[Y]$

$$= 0,4 - (0,7)(0,55) = 0,015 = \frac{3}{200} \rightarrow \checkmark$$

$$E[X] = 0 \cdot 0,3 + 1 \cdot 0,7 = 0,7 \checkmark$$

$$E[Y] = -1 \cdot 0,3 + 1 \cdot 0,55 + 2 \cdot 0,15 = 0,55 \checkmark$$

$$E[XY] = \sum_{y \in S_Y} \sum_{x \in S_X} x y \cdot P_{X,Y}(x,y)$$

$$= 0 \cdot (-1) \cdot 0,1 + 0 \cdot 1 \cdot 0,15 + 0 \cdot 2 \cdot 0,05 \\ + 1 \cdot (-1) \cdot 0,2 + 1 \cdot 1 \cdot 0,4 + 1 \cdot 2 \cdot 0,1$$

$$= 0,4 \checkmark$$

↓ Process ✓