Tutorial Test 5 - Tutoriaal Toets 5 (20 minutes)

Name and surname / Naam en van	Memo	**Mark / Punt [12]**
Student number / Studentenommer		12

Mini formula sheet / Mini-formuleblad		
$\mathrm{E}[g(X)] = \int g(x) f_X(x)\,dx$ $\mathrm{Var}[X] = \mathrm{E}[X^2] - \big(\mathrm{E}[X]\big)^2$	$\mathrm{P}(a \le X \le b) = \int_a^b f_X(x)\,dx$	$\mathrm{P}(X > t) = 1 - F_X(t)$

Question / Vraag 1 [7] 7

A continuous random variable X has PDF

$$f_X(x) = \begin{cases} c(1-x^2), & -1 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a) Prove that $c = \frac{3}{4}$. [2]

b) Calculate $\mathrm{P}(|X| \le 1/2)$. [2]

'n Kontinue toevalsveranderlike X het die WDF wat hieronder gegee word.

$$f_X(x) = \begin{cases} c(1-x^2), & -1 \le x \le 1, \\ 0, & \text{andersins.} \end{cases}$$

a) Bewys dat $c = \frac{3}{4}$. [2] 2

b) Bereken $\mathrm{P}(|X| \le 1/2)$. [2] 2

a) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$ ✓

$\int_{-1}^{1} c(1-x^2)\,dx = 1$

$c\left[x - \frac{1}{3}x^3\right]_{-1}^{1} = 1$ ✓ Process

$c\left[1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right)\right] = 1$

$c\left[2 - \frac{2}{3}\right] = 1$

$c\left(\frac{4}{3}\right) = 1 \Rightarrow c = \frac{3}{4}$ (no mark since given)

b) $P(|X| \le \frac{1}{2}) = P(-\frac{1}{2} \le X \le \frac{1}{2})$ ✓

$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{3}{4}(1-x^2)\,dx$

$= \frac{11}{16}$ ✓

c) Use symmetry to state E[X], and then calculate Var[X]. [3]

c) Gebruik simmetrie om E[X] te gee, en bereken dan Var[X]. [3] 3

c) $E[X] = 0$ ✓ (By symmetry)

$$Var[X] = E[X^2] - E[X]^2 = E[X^2] = \int_{-1}^{1} x^2 \cdot \frac{3}{4}(1-x^2)\,dx$$ ✓

$$= \frac{3}{4}\int_{-1}^{1} x^2 - x^4\,dx = \frac{3}{4}\left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_{-1}^{1}$$

$$= \frac{3}{4}\left[\frac{1}{3} - \frac{1}{5} - \left(-\frac{1}{3} + \frac{1}{5}\right)\right]$$

$$= \frac{3}{4}\left[\frac{2}{3} - \frac{2}{5}\right] = \frac{1}{5} = 0{,}2$$ ✓

Calculator can go directly

Question / Vraag 2 [5] 5

A network delay D is zero with probability 0.2 (instant delivery). Otherwise, with probability 0.8, it is uniformly distributed between 0 and 4 seconds.

a) Make a rough sketch of the PDF, indicating all relevant values on the axes. [2]

b) Give the complete **piecewise** definition of the CDF $F_D(d)$. [2]

c) Calculate P($D > 3$). [1]

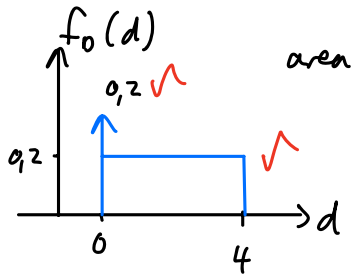
'n Netwerkvertraging D het 'n puntmassa van 0.2 by nul. Dit verteenwoordig onmiddellike aflewering. Met waarskynlikheid 0.8 is die vertraging uniform tussen 0 en 4 sekondes versprei.

a) Teken 'n rofweg skets van die PDF met al die relevante waardes op die asse aangedui. [2] 2

b) Bepaal die volledige **stukgewys** definisie van die KVF $F_D(d)$. [2] 2

c) Bereken P($D > 3$). [1] 1

a)



area = w.h ⇒ $h = \frac{area}{w} = \frac{0{,}8}{4} = 0{,}2$ ✓

b)
$$F_D(d) = \begin{cases} 0 & d < 0 \\ 0{,}2\,u(d) + 0{,}2d & 0 \le d \le 4 \\ 1 & d > 4 \end{cases}$$ ✓ ✓ ✓

c) $P(D > 3) = 1 - P(D \le 3)$

$= 1 - F_D(3)$

$= 1 - (0{,}2 + 0{,}2(3))$

$= 0{,}2 = \frac{1}{5}$ ✓