

Tutorial Test 3 - Tutoriaal Toets 3 (15 minutes)

Name and surname / Naam en van	Memo	Mark / Punt [10]
Student number / Studentenommer		10

Mini formula sheet / Mini-formuleblad		
$E[X] = \sum_{x \in S_X} x P_X(x)$	$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx$	$\text{Var}[X] = E[X^2] - (E[X])^2$
$E[aX + b] = a E[X] + b$	$\text{Var}[aX + b] = a^2 \text{Var}[X]$	$\text{Cov}[X, Y] = E[XY] - E[X]E[Y]$

The discrete random variable X has PMF

$$P_X(x) = \begin{cases} c, & x = -2, \\ 2c, & x = 0, \\ c, & x = 2, \\ 0, & \text{otherwise.} \end{cases}$$

Die diskrete toevalsveranderlike X het die gegewe WMF.

$$P_X(x) = \begin{cases} c, & x = -2, \\ 2c, & x = 0, \\ c, & x = 2, \\ 0, & \text{andersins.} \end{cases}$$

- a) Determine c . [2]
 b) Write the CDF $F_X(x)$ as a complete piecewise expression. [3]
 c) Calculate $E[X]$ and $\text{Var}[X]$. [3]
 d) Now define $Y = X^2$. Determine the PMF $P_Y(y)$. Account for every value of X . [2]

- a) Bepaal c . [2]
 b) Skryf die KVF $F_X(x)$ as 'n volledige stuksgewyse uitdrukking. [3]
 c) Bereken $E[X]$ en $\text{Var}[X]$. [3]
 d) Definieer nou $Y = X^2$. Bepaal die WMF $P_Y(y)$. Neem elke waarde van X in ag. [2]

a) $\sum_{x \in S_X} P_X(x) = 1$ ✓

$$c + 2c + c = 1$$

$$4c = 1$$

$$c = \frac{1}{4}$$

→

mark the rest
with mistake if $c \neq \frac{1}{4}$

b) $F_X(x) = \begin{cases} 0 & x < -2 \quad \checkmark \\ \frac{1}{4} & -2 \leq x < 0 \quad \checkmark \\ \frac{3}{4} & 0 \leq x < 2 \quad \checkmark \\ 1 & x \geq 2 \quad \checkmark \end{cases}$

values intervals

$$\begin{aligned}
 c) E[X] &= \sum_{x \in S_X} x \cdot P_X(x) \\
 &= -2 \cdot \left(\frac{1}{4}\right) + 0 \cdot \left(\frac{1}{2}\right) + 2 \cdot \left(\frac{1}{4}\right) \\
 &= 0 \quad \checkmark
 \end{aligned}$$

$$\text{Var}[X] = E[X^2] - E[X]^2$$

$$\begin{aligned}
 E[X^2] &= \sum_{x \in S_X} x^2 \cdot P_X(x) \\
 &= (-2)^2 \cdot \left(\frac{1}{4}\right) + 0^2 \cdot \left(\frac{1}{2}\right) + 2^2 \cdot \left(\frac{1}{4}\right) \\
 &= 1 + 0 + 1 \\
 &= 2 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \text{Var}[X] &= 2 - (0)^2 \\
 &= 2 \quad \checkmark \\
 &\quad \longrightarrow
 \end{aligned}$$

d) $Y = X^2$ from $S_X = \{-2, 0, 2\}$
 $S_Y = \{0, 4\}$

X	-2	0	2
$Y = X^2$	4	0	4

$$P_Y(y) = \begin{cases} \frac{1}{2} & y=0 \\ \frac{1}{2} & y=4 \\ 0 & \text{otherwise} \end{cases} \quad \checkmark \text{ added probabilities. or } \begin{cases} \frac{1}{2} & y=0 ; y=4 \\ 0 & \text{otherwise} \end{cases} \quad \checkmark \text{ everything else correct.}$$