Tutorial Test 2 - Tutoriaal Toets 2 (18 minutes)

Name and surname / Naam en van	Memo	**Mark / Punt [11]**
Student number / Studentenommer		

Mini formula sheet / Mini-formuleblad		
$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)},\ \mathrm{P}(B) > 0$ $\mathrm{P}(A\cap B) = \mathrm{P}(A\mid B)\,\mathrm{P}(B)$	$\mathrm{P}(\overline{A}) = 1 - \mathrm{P}(A)$ $\mathrm{P}(A\cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A\cap B)$	$\mathrm{P}(A) = \sum_{i=1}^{m} \mathrm{P}(A\mid B_i)\,\mathrm{P}(B_i)$ $\{B_i\}_{i=1}^{m}$ partition / partisie, $\mathrm{P}(B_i) > 0$
$\mathrm{P}(B\mid A) = \dfrac{\mathrm{P}(A\mid B)\,\mathrm{P}(B)}{\mathrm{P}(A)},\ \mathrm{P}(A) > 0$	$P_k^n = (n)_k = \dfrac{n!}{(n-k)!}$ $\dbinom{n}{k} = \dfrac{n!}{(n-k)!\,k!}$	$\mathrm{P}(E) = \dfrac{n(E)}{n(S)}$ finite, equally likely / eindig, ewe waarskynlik

Question / Vraag 1 [6]

A service centre tests returned motor controllers. For a randomly selected controller, let F denote an actual hardware fault and let $N = \overline{F}$ denote normal operation. Let A denote that the automated bench test displays a red fault warning. The supplied probabilities mean:

- $\mathrm{P}(F) = 0.2$: the actual hardware-fault rate among returned controllers is 20%.
- $\mathrm{P}(A \mid F) = 0.9$: the test warns for 90% of faulty controllers.
- $\mathrm{P}(A \mid N) = 0.1$: the test warns for 10% of normal controllers; this is a false alarm.

a) Calculate $\mathrm{P}(A)$. [3]

b) Given that the test displays the red fault warning, calculate $\mathrm{P}(F \mid A)$. [3]

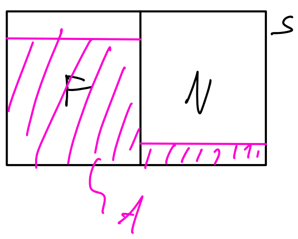
'n Dienssentrum toets terugbesorgde motorbeheerders. Vir 'n ewekansig gekose beheerder, laat F 'n werklike hardewarefout aandui en laat $N = \overline{F}$ normale werking aandui. Laat A aandui dat die outomatiese toetstelsel 'n rooi foutwaarskuwing vertoon. Die gegewe waarskynlikhede beteken:

- $\mathrm{P}(F) = 0.2$: die werklike hardewarefoutkoers onder terugbesorgde beheerders is 20%.
- $\mathrm{P}(A \mid F) = 0.9$: die toets waarsku vir 90% van foutiewe beheerders.
- $\mathrm{P}(A \mid N) = 0.1$: die toets waarsku vir 10% van normale beheerders; dit is 'n vals alarm.

a) Bereken $\mathrm{P}(A)$. [3]

b) Gegee dat die toets die rooi foutwaarskuwing vertoon, bereken $\mathrm{P}(F \mid A)$. [3]

a) Use the Law of Total probability



$$P(A) = P(A|F)P(F) + P(A|N)P(N)$$

$$= 0.9 \cdot 0,2 + 0,1 \cdot (1-0,2)$$

$$= \frac{13}{50} = 0,26$$

b) $$P(F|A) = \frac{\overset{P(A\cap F)}{P(A|F)P(F)}}{P(A)}$$ (Bayes' theorem)

$$= \frac{0.9 \cdot 0,2}{0,26} = \frac{9}{13} = 0,6923\ldots$$

Question / Vraag 2 [5]

Six distinct sensor modules are available. One is assigned as primary and a different one as backup. Two of the six modules are high-grade.

a) How many ordered primary–backup assignments are possible? [2]

b) If an assignment is selected uniformly at random, calculate the probability that both assigned modules are high-grade. [3]

Ses verskillende sensormodules is beskikbaar. Een word as primêr en 'n ander een as rugsteun toegeken. Twee van die ses modules is hoëgraad-modules.

a) Hoeveel geordende primêr–rugsteuntoewysings is moontlik? [2]

b) Indien 'n toewysing uniform ewekansig gekies word, bereken die waarskynlikheid dat albei toegewysde modules hoëgraadmodules is. [3]

a) ordered = sequence → Permutations ✓

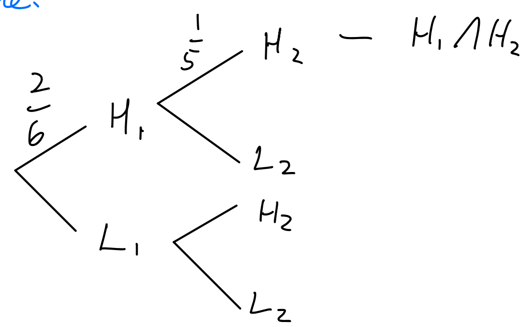
$$P^6_2 = \frac{6!}{(6-2)!} = 30 \checkmark$$

b) $P(H_1 \cap H_2)$ → Both high grade.

$$= P(H_1) \cdot P(H_2 | H_1)$$

$$= \frac{2}{6} \checkmark \cdot \frac{1}{5} \checkmark$$

$$= \frac{2}{30} = \frac{1}{15} = 0,0\dot{6} \checkmark$$



$$\left(\text{or} \quad P(H_1 \cap H_2) = \frac{n(H_1 \cap H_2)}{n(S)} = \frac{P^2_2}{P^6_2} = \frac{2}{30} = \frac{1}{15} \right)$$

(or since order of High-grade doesn't matter:

$$\frac{1}{\binom{6}{2}} = \frac{1}{15}$$)