

Tutorial Test 12 - Tutoriaal Toets 12 (15 minutes)

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Consider the voltage process below. $X(t)$ is a zero-mean WSS Gaussian process. The Gaussian random offset A has $\mu_A = 0$ and $\sigma_A = 1$, is independent of the entire process $X(t)$, and stays constant throughout each recording. Voltages are in volts and time is in seconds.

$$Y(t) = X(t) + A, \quad R_X(\tau) = 4e^{-2|\tau|}, \quad t \in \mathbb{R}.$$

Calculate the mean μ_Y and autocorrelation $R_Y(t, \tau) = E[Y(t)Y(t + \tau)]$. Is $Y(t)$ wide-sense stationary (WSS)? Justify your conclusion. [6]

Beskou die spanningsproses hieronder. $X(t)$ is 'n nulgemiddelde WSS Gaussiese proses. Die Gaussiese ewekansige afset A het $\mu_A = 0$ en $\sigma_A = 1$, is onafhanklik van die hele proses $X(t)$, en bly gedurende elke opname konstant. Spannings is in volt en tyd is in sekondes.

Bereken die gemiddelde μ_Y en outokorrelasie $R_Y(t, \tau) = E[Y(t)Y(t + \tau)]$. Is $Y(t)$ wydsin-stasionêr (WSS)? Motiveer jou gevolgtrekking. [6]

$$\begin{aligned} \mu_Y(t) &= E[Y(t)] = E[X(t) + A] \\ &= E[X(t)] + E[A] \\ &= 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} R_Y(t, \tau) &= E[Y(t)Y(t + \tau)] \\ &= E[(X(t) + A)(X(t + \tau) + A)] \\ &= E[X(t)X(t + \tau) + AX(t) + AX(t + \tau) + A^2] \\ &= E[X(t)X(t + \tau)] + E[AX(t)] + E[AX(t + \tau)] + E[A^2] \end{aligned}$$

Process ✓

- Since $X(t)$ and A are independent $E[AX(t)] = E[A]E[X(t)] = 0$ ✓
- $\text{Var}[A] = E[A^2] - E[A]^2 \Rightarrow \text{Var}[A] = E[A^2] = 1$ ✓

$$\begin{aligned} \therefore R_Y(t, \tau) &= R_X(\tau) + 0 + 0 + E[A^2] \\ &= 4e^{-2|\tau|} + 1 = R_Y(\tau) \quad \checkmark \end{aligned}$$

Since μ_Y and $R_Y(\tau)$ are time invariant $Y(t)$ is WSS. ✓ with reason

