

Tutorial Test 11 - Tutoriaal Toets 11 18 minutes

Name and surname / Naam en van	Memo	Mark / Punt [18] 9
Student number / Studentenommer		
Mini formula sheet / Mini-formuleblad		
U : Uniform $[a, b]$ $E[U] = \frac{a+b}{2}$	U : Uniform $[a, b]$ $\text{Var}[U] = \frac{(b-a)^2}{12}$	$\text{Var}[cU] = c^2 \text{Var}[U]$ c : constant / konstant

During a calibration run, a sensor's response is modelled by

Gedurende 'n kalibrasieoplopie word 'n sensor se respons gemodelleer deur

$$X(t) = A(1+t), \quad t \geq 0.$$

Time is measured in seconds. The random response coefficient A is uniform on $[1, 2]$. One value of A is drawn at the start of each run and remains fixed throughout that run; it is not redrawn at each time.

Tyd word in sekondes gemeet. Die ewekansige responskoeffisiënt A is uniform op $[1, 2]$. Een waarde van A word aan die begin van elke lopie getrek en bly deur daardie hele lopie vas; dit word nie op elke tydstip opnuut getrek nie.

a) In one particular run, the output measured after one second ($t = 1$) is $X(1) = 3$. Predict the output two seconds later. Show your working. [3]

a) In een bepaalde lopie is die uitset wat na een sekonde ($t = 1$) gemeet word, $X(1) = 3$. Voorspel die uitset twee sekondes later. Toon jou berekening. [2]

b) In general, calculate the probability that the output exceeds 5 after two seconds: $P(X(2) > 5)$. [2]

b) Bereken in die algemeen die waarskynlikheid dat die uitset na twee sekondes 5 oorskry: $P(X(2) > 5)$. [2]

c) Let $D = X(2) - X(0)$ be the increase in response between $t = 0$ and $t = 2$. Calculate $E[D]$ and $\text{Var}[D]$. Show your working. [3]

c) Laat $D = X(2) - X(0)$ die toename in respons tussen $t = 0$ en $t = 2$ wees. Bereken $E[D]$ en $\text{Var}[D]$. Toon jou berekening. [3]

d) Let $Y_1 = X(1) - X(0)$ and $Y_2 = X(2) - X(1)$ be the output increases during the first and second one-second intervals, respectively. Are Y_1 and Y_2 independent? Justify your conclusion. [3]

d) Laat $Y_1 = X(1) - X(0)$ en $Y_2 = X(2) - X(1)$ die uitsettoenames tydens onderskeidelik die eerste en tweede eensekonde-interval wees. Is Y_1 en Y_2 onafhanklik? Motiveer jou gevolgtrekking. [2]

a) Two seconds later = $1+2 = 3$ s

$$X(3) = a(1+3) = 4a$$

$$\text{Given: } X(1) = a(1+1) = 2a = 3 \Rightarrow a = \frac{3}{2} \checkmark$$

$$\therefore X(3) = 4\left(\frac{3}{2}\right) = 6 \checkmark$$

$$\begin{aligned}
 b) \quad P(X(2) > 5) &= P(A(1+2) > 5) = P(A > \frac{5}{3}) \stackrel{\checkmark}{=} 1 - P(A \leq \frac{5}{3}) \\
 &\stackrel{\checkmark}{=} 1 - F_A(\frac{5}{3}) \\
 &= \int_{\frac{5}{3}}^2 1 \, da \\
 &= 2 - \frac{5}{3} = \frac{1}{3} \checkmark \\
 &\quad \rightarrow
 \end{aligned}$$

$$z) \quad D = X(2) - X(0)$$

$$\begin{aligned}
 E[D] &= E[X(2) - X(0)] \\
 &= E[A(1+2) - A(1+0)] \\
 &= E[2A] \\
 &= 2E[A] = 2\left(\frac{1+2}{2}\right) \\
 &= 3 \checkmark \\
 &\quad \rightarrow
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}[D] &= \text{Var}[2A] \\
 &= 2^2 \text{Var}[A] \checkmark \\
 &= 4 \left(\frac{(2-1)^2}{12} \right) \\
 &= \frac{1}{3} \checkmark \\
 &\quad \rightarrow
 \end{aligned}$$

$$d) \quad Y_1 = X(1) - X(0) = A(1+1) - A(1+0) = A$$

$$Y_2 = X(2) - X(1) = A(1+2) - A(1+1) = A$$

Since $Y_1 = Y_2$ they are not independent [✓], knowing the one fixes the other. ^{✓ reason}

$$\text{Also } \text{Cov}[Y_1, Y_2] = \text{Cov}[A, A] = \text{Var}[A] = \frac{1}{12} \neq 0$$