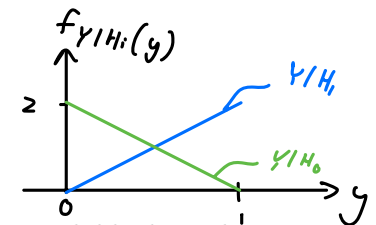


Tutorial Test 10 - Tutoriaal Toets 10 (²⁰~~15~~ minutes)

Name and surname / Naam en van	Memo	Mark / Punt [11]
Student number / Studentenommer		
Mini formula sheet / Mini-formuleblad		
ML : $\frac{f_{Y H_0}(y)}{f_{Y H_1}(y)} \geq 1$	MAP : $\frac{f_{Y H_0}(y)}{f_{Y H_1}(y)} \geq \frac{P(H_1)}{P(H_0)}$	$P_{ERR} = P(H_0)P_{FA} + P(H_1)P_{MISS}$

A motor is assessed using a continuous, normalised vibration index Y . The hypotheses are H_0 : healthy motor and H_1 : worn bearing, with $P(H_0) = 0.70$ and $P(H_1) = 0.30$. The conditional PDFs are given below; a worn bearing tends to produce a higher reading.

y	$f_{Y H_0}(y)$	$f_{Y H_1}(y)$
$0 \leq y \leq 1$	$2(1 - y)$	$2y$
$y \notin [0, 1]$	0	0



For the ML and MAP inequalities in the formula sheet, choose H_0 when the inequality holds and H_1 otherwise. Let A_i be the set of readings for which H_i is chosen.

a) Determine the ML decision regions A_0 and A_1 , giving each as a range of y -values. Show how you obtain the numerical boundary between them. [3]

b) Determine the MAP decision regions A_0 and A_1 . Show how you obtain their numerical boundary. [3]

c) For the MAP rule, calculate the conditional false-alarm probability (choose H_1 when H_0 is true), the conditional miss probability (choose H_0 when H_1 is true), and the overall probability of error. Show the probability formulation and calculation for each conditional error. [5]

'n Motor word beoordeel met 'n kontinue, genormaliseerde vibrasie-indeks Y . Die hipoteses is H_0 : gesonde motor en H_1 : verslete laer, met $P(H_0) = 0.70$ en $P(H_1) = 0.30$. Die voorwaardelike WDFs word hieronder gegee; 'n verslete laer lewer gewoonlik 'n hoër lesing.

Vir die MA- en MAP-ongelykhede in die formuleblad, kies H_0 wanneer die ongelykheid geld en andersins H_1 . Laat A_i die stel lesings wees waarvoor H_i gekies word.

a) Bepaal die MA-besluitgebiede A_0 en A_1 en gee elkeen as 'n interval van y -waardes. Toon hoe jy die numeriese grens tussen hulle bepaal. [3]

b) Bepaal die MAP-besluitgebiede A_0 en A_1 . Toon hoe jy hulle numeriese grens bepaal. [3]

c) Bereken vir die MAP-reël die voorwaardelike valsalarmwaarskynlikheid (kies H_1 wanneer H_0 waar is), die voorwaardelike miswaarskynlikheid (kies H_0 wanneer H_1 waar is), en die algehele foutwaarskynlikheid. Toon die waarskynlikheidsformulering en berekening vir elke voorwaardelike fout. [5]

$$\begin{aligned}
 a) \quad \frac{f_{Y|H_0}(y)}{f_{Y|H_1}(y)} &\geq 1 \Rightarrow f_{Y|H_0}(y) \geq f_{Y|H_1}(y) \\
 2(1-y) &\geq 2y \\
 2 &\geq 4y \\
 0.5 &\geq y \quad \text{then choose } H_0
 \end{aligned}$$

$$\begin{aligned}
 \therefore A_0 &= [0, 0.5] & A_1 &= (0.5, 1] \\
 &= 0 \leq y \leq 0.5 & &= 0.5 < y \leq 1
 \end{aligned}$$

Process ✓

$$b) \frac{f_{Y|H_0}(y)}{f_{Y|H_1}(y)} \geq \frac{P(H_1)}{P(H_0)} = \frac{0,3}{0,7} = \frac{3}{7} \approx 0,43$$

$$\Rightarrow 2(1-y) \geq \frac{3}{7} \cdot 2y$$

$$2 \geq \frac{3}{7} y$$

$$0,7 \geq y$$

$$\therefore A_0 = [0, 0,7]$$

$$= \{0 \leq y \leq 0,7\}$$

$$A_1 = (0,7, 1]$$

$$= \{0,7 < y \leq 1\}$$

Process

$$z) P_{FA} = P(A_1 | H_0) \quad (\text{given})$$

$$= P(0,7 < y \leq 1 | H_0)$$

$$= \int_{0,7}^1 2(1-y) dy$$

$$= \frac{1}{100} = \underline{0,01}$$

$$P_{MISS} = P(A_0 | H_1)$$

(given)

$$= P(0 \leq y \leq 0,7 | H_1)$$

$$= \int_0^{0,7} 2y dy$$

$$= \frac{49}{100} = \underline{0,49}$$

$$P_{ERR} = P(H_0) P_{FA} + P(H_1) \cdot P_{MISS}$$

(given)

$$= 0,7 \cdot 0,01 + 0,3 \cdot 0,49$$

$$= \frac{21}{100} = \underline{0,21}$$