Tutorial Test 4 - Tutoriaal Toets 4 (20 minutes)

Name and Surname	Memo	Mark [14]
Student number		14

Question 1 [8]

In a *Fortnite* tournament, 50 players each independently have a 2% chance of finding a legendary weapon in the first supply drop. Let X be the total number of legendary weapons found.

a) Identify the probability distribution of X and state its parameters. [2]

X is a Binomial $(n=50,\ p=0{,}02)$ Random Variable.

(or binomial PMF shown) (half for each).

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b) Calculate the probability that no legendary weapons are found in the first supply drop. [3]

$$P(X=0) = \binom{n}{x} p^x (1-p)^{n-x}$$

$x=0$

$$= \binom{50}{0} 0{,}02^0 (0{,}98)^{50}$$

$$= 1 \cdot 1 \cdot 0{,}98^{50}$$ Values sub.

$$\simeq 0{,}36417$$

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c) The game developers want to simulate millions of matches on their servers to balance loot drop rates. Computing the true values repeatedly is expensive, so they propose using the *Poisson* approximation instead. Using this approach:

(i) Compute $P(X=0)$

(ii) Calculate the percentage error relative to the exact probability from part (b). [3]

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$$P_X(x) = \frac{\alpha^x e^{-\alpha}}{x!}$$ (✓ for formula)

$E[X] = \alpha$ (average drop rate per match)

$\alpha = 0{,}02 \cdot 50 = 1$

i) $$P(X=0) = P_X(0) = \frac{1^0 e^{-1}}{0!} = e^{-1} \simeq 0{,}36788$$

ii) $$\%\ \text{error} = \left| \frac{0{,}98^{50} - e^{-1}}{0{,}98^{50}} \right| = 0{,}0101869\ldots \simeq 1{,}02\%$$

Vraag 1 [8] 8

In 'n *Fortnite*-toernooi het 50 spelers elk onafhanklik 'n kans van 2% om 'n legendariese wapen in die eerste voorsieningsval te kry. Laat X die totale aantal legendariese wapens wees wat gevind word.

a) Identifiseer die waarskynlikheidsverdeling van X en gee sy parameters. [2]

b) Bereken die waarskynlikheid dat geen legendariese wapens in die eerste voorsieningsval gevind word nie. [3]

c) Die spelontwikkelaars wil miljoene wedstryde op hul bedieners simuleer om buitvalkoerse te balanseer. Om die presiese waardes herhaaldelik te bereken is duur, daarom stel hulle voor om die *Poisson*-benadering te gebruik. Gebruik hierdie metode:

(i) Bereken $P(X=0)$

(ii) Bereken die persentasiefout relatief tot die presiese waarskynlikheid in deel (b). [3]

Question 2 [6] / Vraag 2 [6]

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Let X be a continuous random variable with PDF:

$$f_X(x) = \begin{cases} k\,x, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Laat X 'n aaneenlopende toevalsveranderlike wees met KWF:

$$f_X(x) = \begin{cases} k\,x, & 0 \le x \le 2, \\ 0, & \text{andersins.} \end{cases}$$

a) Find k. [2] **a)** Vind k. [2]

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$\int_0^2 kx\,dx = 1$$

$$\frac{1}{2}kx^2\Big|_0^2 = \frac{k}{2}[4-0]$$

$$2k = 1$$

$$k = \frac{1}{2}$$

b) Derive the CDF $F_X(x)$. [2] **b)** Lei die KVF $F_X(x)$ af. [2]

for $0 \le x \le 2$

$$F_X(x) = \int_0^x \frac{t}{2}\,dt$$

$$= \frac{t^2}{4}\Big|_0^x = \frac{x^2}{4}$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

c) Compute $P(0.5 \le X \le 1.5)$. [2] **c)** Bereken $P(0.5 \le X \le 1.5)$. [2]

$$P(0{,}5 \le X \le 1{,}5) = F_X(1{,}5) - F_X(0{,}5)$$

$$= \frac{(1.5)^2}{4} - \frac{(0{,}5)^2}{4}$$

$$= 0{,}5$$

approach

Or

$$= \int_{0{,}5}^{1{,}5} \frac{x}{2}\,dx$$

$$= \frac{x^2}{4}\Big|_{0{,}5}^{1{,}5}$$

$$= 0{,}5$$

PMFs / WMF's and PDFs / WDF's

Type/Tipe	$P_X(x)$	$\mu_X,\ \sigma_X^2$	Limit/Beperk.
Binomial (n,p)	$\binom{n}{x}p^x(1-p)^{n-x}$	$np,\ np(1-p)$	$0 \le x \le n$
Geometric (p)	$p(1-p)^{x-1}$	$1/p,\ (1-p)/p^2$	$x = 1, 2, 3, \ldots$
Pascal (k,p)	$\binom{x-1}{k-1}p^k(1-p)^{x-k}$	$k/p,\ k(1-p)/p^2$	$x = k, k+1, \ldots$ and $k \ge 1$
Discrete Unif. (k,l)	$1/(l-k+1)$	$(k+l)/2,\quad (l-k)(l-k+2)/12$	$x = k, k+1, \ldots, l$ and $k < l$
Poisson (α)	$\alpha^x e^{-\alpha}/x!$	$\alpha,\quad \alpha$	$x = 0, 1, 2, \ldots$ and $\alpha > 0$