Systems and Signals 344

2026

Lecture 1

Module Introduction and The Mathematics of Probability

Section 1.1

About me

Prof Rensu Theart

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- **Office:** E3020

My research

- 3D microscopy image visualisation and analysis
- Virtual Reality applications
- Machine learning with a focus on deep neural networks applied to image processing problems
    - Human Pose-Estimation
    - Face Recognition
    - Instance segmentation
- Internet of Things (IoT) applications

Welcome to **Systems and Signals 344 (Stochastic Signals)**

From Yearbook:

> *One- and multi-dimensional random variables; expected values, moments, distribution functions and probability density functions; operations on and transformations of random variables; random signals, auto- and cross-correlation, stationary and spectral characteristics; behaviour with linear systems.*

Course Information

- **Language Policy:** Section 7.1.4
    - All information is conveyed in *English*.
    - A brief *Afrikaans summary* is given during each lecture.
    - In these modules, questions asked in contact sessions are answered in the language of the question.

Course Information

- **SAQA Credits:** 15
- **Workload:** 12 hours per week
  - 3 hours Lectures
  - 3 hours Practicals and Tutorials
  - 6 hours Home Study

Course Information

- **Lectures:**
  - Monday 8:00 (E1029)
  - Wednesday 9:00 (E2005)
  - Thursday 12:00 (E1029)
- **Tutorials/Practicals:**
  - Wednesdays 14:00-17:00 (K302/M3002)
  - I will inform you in advance which venue to use (but generally K302)
  - *Tutorial test* every week

Course Information

Final Mark Formula

$$FM = (15\% \times AF) + (35\% \times A1) + (50\% \times A2)$$

AF = Average of Tutorial Tests

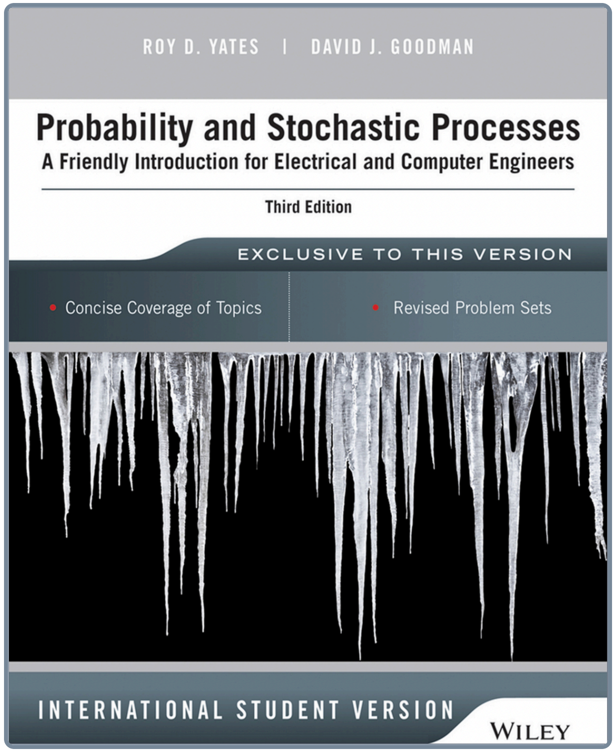
Course Information

- **Assessments**
  - Closed book
  - Calculators allowed
    - Only *pocket calculators* approved by the Engineering Faculty for first- and second-year tests and examinations—i.e. the Sharp EL-W506, EL-506W and Casio FX-991—may be used during assessments.

Textbook

Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers (3rd Edition)

Yates & Goodman



Contact Me?

- **Only** email me about personal issues, or if you have a valid excuse for not attending a Tutorial/Practical.
- Generally, please speak to me after lectures or during Tutorial/Practical sessions.
- Note that there is no Teams channel or STEMLearn Forum for the module.

General

- **I don't use STEMLearn, access the content here.**
- Please engage in class, ask questions, tell me if I make mistakes (I'm human).
- Make hand-written notes during class.
  - Writing by hand tends to boost your ability to retain information, comprehend new ideas, and be more productive.
- You must take ownership of your own learning.
  - Please read ahead in the textbook.
  - Don't fall behind - if you don't understand everything by the end of a Tutorial, then stop, catch up immediately.
- We are in this together, let's help each other.

Introduction

- AI is cool
  - ChatGPT
  - Midjourney Feed: https://www.midjourney.com/showcase
  - Self-driving cars
  - etc.

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At the core of all of these systems lies **Probability Theory**.

Probability

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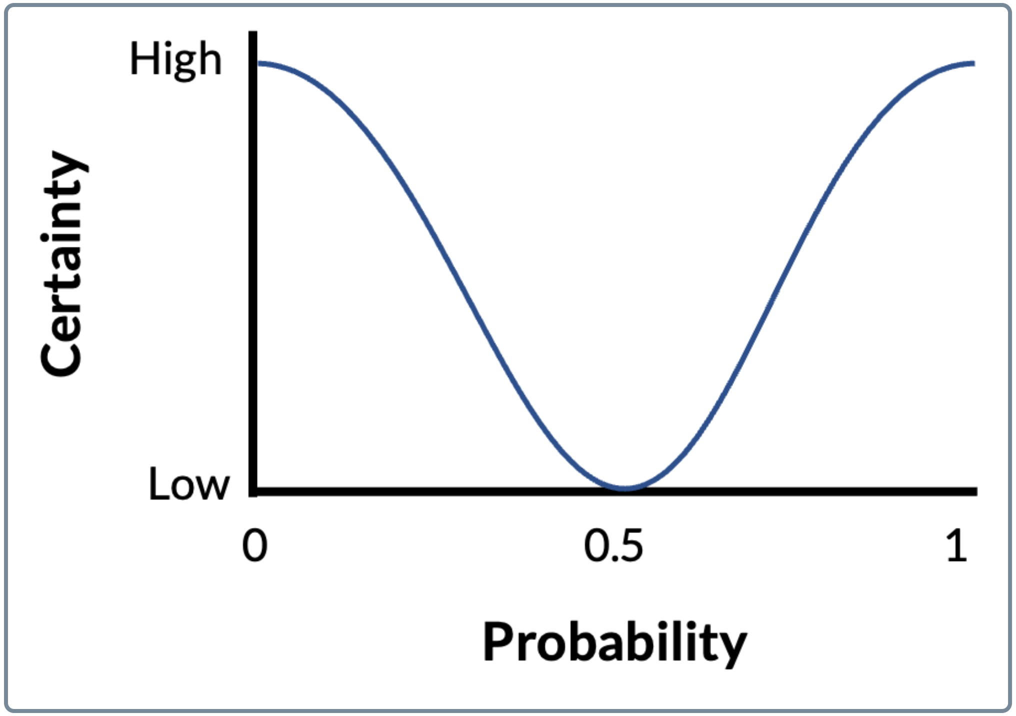
The probability quantifies this "chance" of an event happening and takes values between 0 and 1 inclusive.



Mathematics of Probability

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Uncertainty is due to:

- The situation is so complex that we just can't replicate everything important exactly
- Partial knowledge of the state of the world
- Noisy observations
- Phenomena not covered by our model
- Inherent *stochasticity* of the world
    - Randomness that can be modelled with a *probability distribution*.

Mathematics of Probability

We will look at the abstract *mathematics of probability* which consists of *definitions*, *axioms*, and *theorems* that follow from the axioms.

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- **Definitions** establish the terminology and mathematical objects used in probability theory.
  - The frame of reference and terminology we use.
- **Axioms** are statements accepted as the starting assumptions of a theory.
  - The standard formulation of probability has *three* axioms.
- **Theorems** are consequences that follow logically from definitions and axioms.
  - Each theorem has a *proof* that refers to definitions, axioms, and other theorems.

Definitions/Terminology

Probability is based on a repeatable **experiment** that consists of a *procedure* and *observations* with an associated *model*.

An **outcome** is any possible observation of that experiment.

An **event** is a set of outcomes of an experiment.

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A **random experiment** is an experiment for which the possible outcomes are known, but the outcome of an individual performance cannot be predicted with certainty.

A single performance of a random experiment is a **trial**.

Definitions/Terminology

Sample Space (S): The set of all possible outcomes of an experiment.

Favourable Outcome: An outcome that belongs to the event of interest E, where $E \subseteq S$.

For a finite sample space with equally likely outcomes:

$$\text{Probability}(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}}$$

Definitions/Terminology

Sample Space (S): The set of all possible outcomes of an experiment.

Favourable Outcome: An outcome that belongs to the event of interest E, where $E \subseteq S$.

For a finite sample space with equally likely outcomes:

$$P(E) = \frac{n(E)}{n(S)}$$

Note that E is just a *name* we designate to the event, but it could have been A, B, "even numbers", "heads", etc.

Types of Events

- **Collectively Exhaustive Events:** A collection of events is collectively exhaustive if their union equals the sample space, $\bigcup_i E_i = S$.
- **Certain Event:** The event is certain to occur. $P(E) = 1$
- **Impossible Event:** The empty set is an impossible event. $P(\emptyset) = 0$
- **Equally Likely Events:** Events that have the same probability of occurring.
- **Mutually Exclusive Events:** If they have no outcomes in common.
- **Independent events:** Events that are not affected by other events.
- **Dependent events:** Events that are affected by other events.

Set Theory

The mathematical basis of probability is the theory of sets.

Set Theory

- A **set** is a collection of distinct objects.
- Each object in a set is known as an **element**.
  - E.g. $A = \{1, 2, 3, 4, 5\}$
- If element x is a member of set A then $x \in A$
- If element x is not a member of set A then $x \notin A$
- The **empty set** / **null set** ($\emptyset$) has no elements. $\emptyset = \{\}$
- The **universal set** (S) contains all elements that could possibly be of interest in a particular context.
  - All sets are subsets of S.
  - Sometimes the symbol Ω or $\mathcal{U}$ is used instead.
- If every element of set A is also an element of set B then we say that $A \subseteq B$ (A is a **subset** of B)
- If $A \subseteq B$ and $B \subseteq A$ then $A = B$ (A and B are **equal**)

Set Theory

- **Cardinality**: $n(A)$ or $\#A$ is the number of elements in A
- If set A contains a *finite* number N of elements x_i then we denote
  $$A = \{x_1, x_2, \ldots, x_N\}$$
- If set A contains an *infinite* number of elements x_i that can be enumerated (can be written down in a list) then we denote
  $$A = \{x_1, x_2, \ldots\}$$
  (Such a set is **countably infinite**)
- A set A of all elements satisfying property L is denoted
  $$A = \{x | x \text{ satisfies } L\}$$ (read as "such that")
  For example, the set of all real numbers in the interval $[0, 1]$ is denoted
  $$A = \{x \in \mathbb{R} | 0 \leq x \leq 1\}$$
  (This set is **uncountably infinite**)

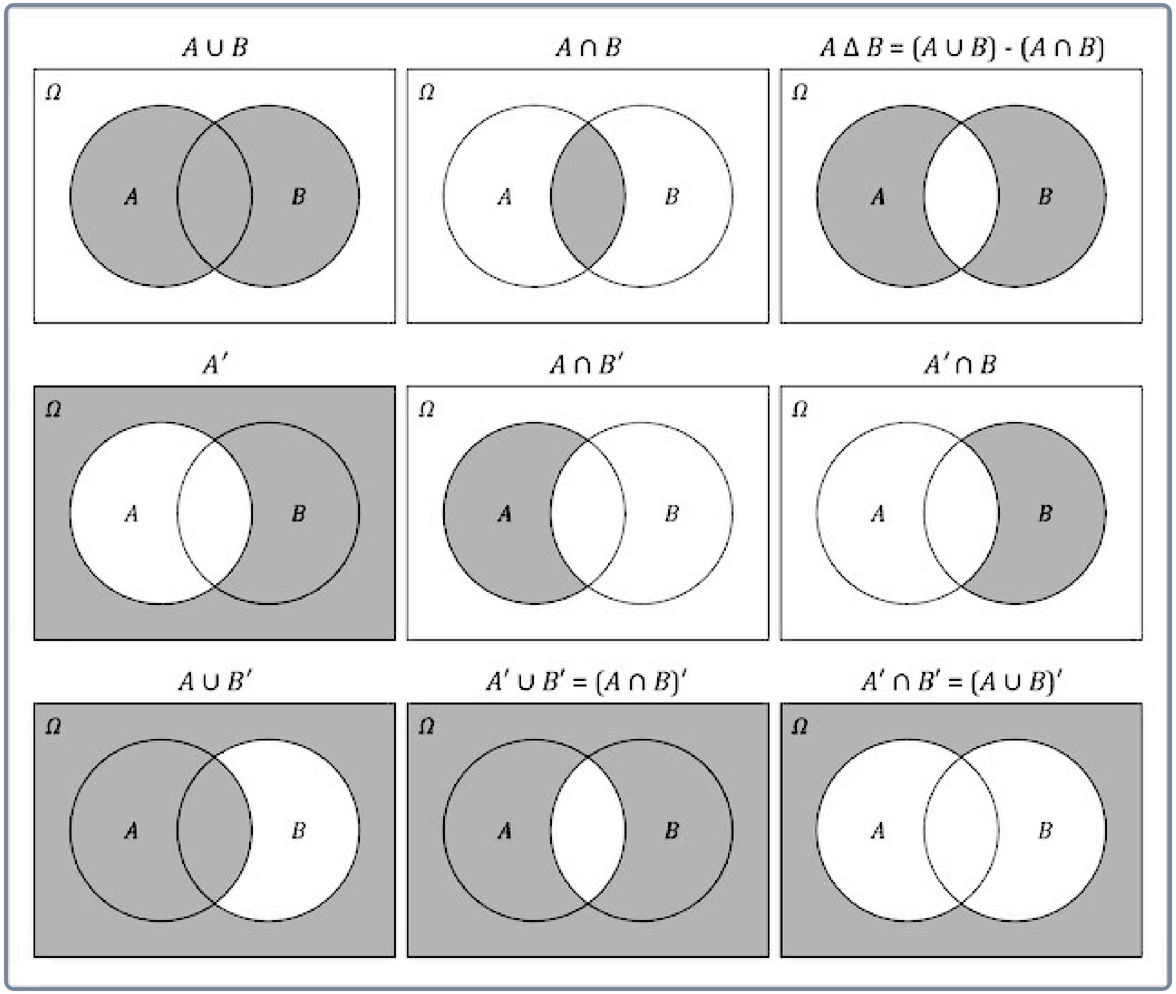
Set Operations

- **Complement (not)**: $\bar{A} = \{x \in S | x \notin A\}$
  (all elements in S not in A)
  - *Note:* $\bar{\bar{A}} = A$
- **Union (or)**: $A \cup B = \{x \in S | x \in A \text{ or } x \in B\}$
  (set of elements in A or in B or in both)
  - *Note:* $A \cup S = S$ and $A \cup \emptyset = A$
- **Intersection (and)**: $A \cap B = \{x \in S | x \in A \text{ and } x \in B\}$
  (set of elements that are members of both A and B)
  - *Note:* $A \cap S = A$ and $A \cap \emptyset = \emptyset$

Set Operations

- The **Cartesian product** $A \times B$ is the set of all ordered pairs whose first component is in A and whose second component is in B: $A \times B = \{(a, b) | a \in A,\ b \in B\}$
- **Difference**: $A - B = A \setminus B = \{x \in S | x \in A \text{ and } x \notin B\}$
  (set of all elements in A that are not also in B)
  - *Note*: $A - B = A \cap \bar{B}$
- **Symmetric Difference** (or **disjunctive union**): $A \Delta B = (A \cup B) - (A \cap B)$
  - *Note*: $A \Delta A = \emptyset$ and $A \Delta \emptyset = A$

Set Operations

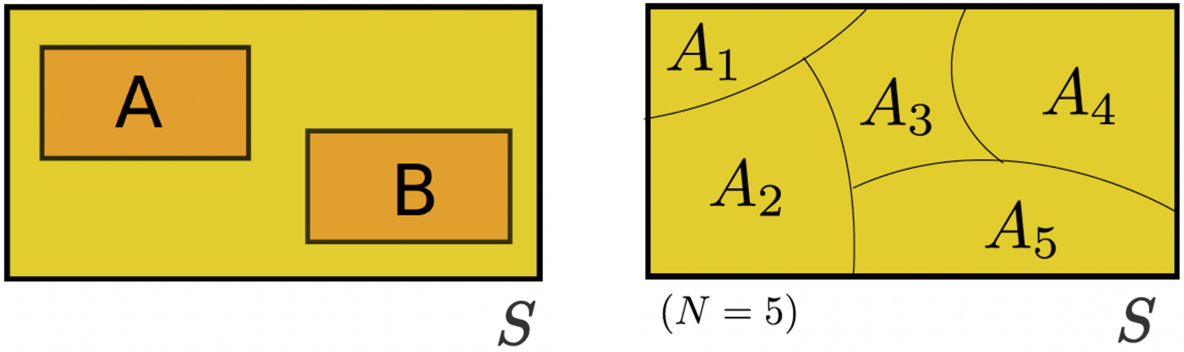


Note that: $\Omega = S$

Set Operations

- A and B are **disjoint** (or **mutually exclusive**) if their intersection is the empty set, i.e. when $A \cap B = \emptyset$
- A collection of sets $A_1, \ldots, A_n$ is a **partition** of B if their union is B and they are **pairwise disjoint**, i.e.

$$\bigcup_{i=1}^{n} A_i = B \quad \text{and} \quad A_i \cap A_j = \emptyset \ \forall \ i \neq j$$



Mutually exclusive

Partition of S

Algebra of Sets

- **Commutative laws**
  - $A \cup B = B \cup A$
  - $A \cap B = B \cap A$
- **Associative laws**
  - $(A \cup B) \cup C = A \cup (B \cup C) = A \cup B \cup C$
  - $(A \cap B) \cap C = A \cap (B \cap C) = A \cap B \cap C$
- **Distributive laws**
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Algebra of Sets

- **Identity laws**
  - $A \cup \emptyset = A$
  - $A \cap S = A$
- **Complement laws**
  - $A \cup \bar{A} = S$
  - $A \cap \bar{A} = \emptyset$

Algebra of Sets

- De Morgan's laws
  - $\overline{A \cup B} = \bar{A} \cap \bar{B}$
  - $\overline{A \cap B} = \bar{A} \cup \bar{B}$
- Cardinality laws (for finite sets)
  - $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
  - $n(A - B) = n(A) - n(A \cap B)$
  - $n(A) + n(\bar{A}) = n(S)$

Set theory and probability

Set Algebra	Probability
Set	Event
Universal set	Sample space
Element	Outcome

Symbols	**Meaning**
$\{\}$	Denotes a set
S or Ω	Universal set
$\emptyset$	Empty set / Null set
$n(X)$ or $\#X$	Cardinality of set X
$x \in A$	x is an element of set A
$x \notin A$	x is not an element of set A
$A \subseteq B$	Set A is a subset of set B
$A \cup B$	Set A union set B (or)
$A \cap B$	Set A intersection set B (and)
$\overline{A}$ or A' or A^c	Complement of set A (not)
$A - B$ or $A \setminus B$	Difference between sets
$A \Delta B$	Symmetric difference