Power Density Spectrum

$$R_x(t,\tau) \overset{WSS}{=} R_x(\tau) = E[X(t)\,X(t+\tau)]$$

$$R_x(0) = E[X^2(t)]$$

$$\text{Power} = W = \frac{J}{s}$$

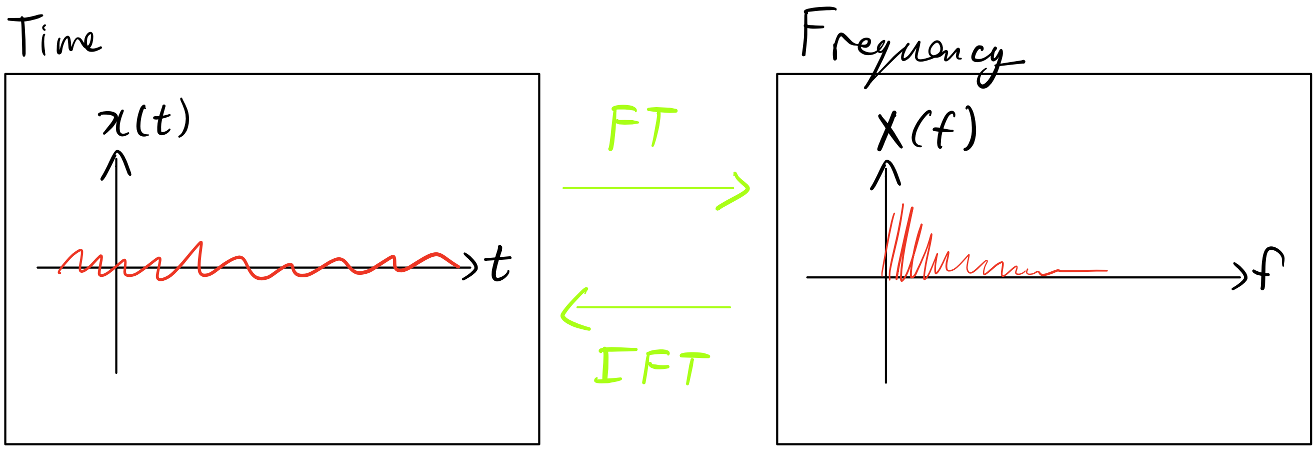
$$\text{Energy} = J$$

$$W \cdot s = J$$

$\Rightarrow$ Integrating Power gives energy.

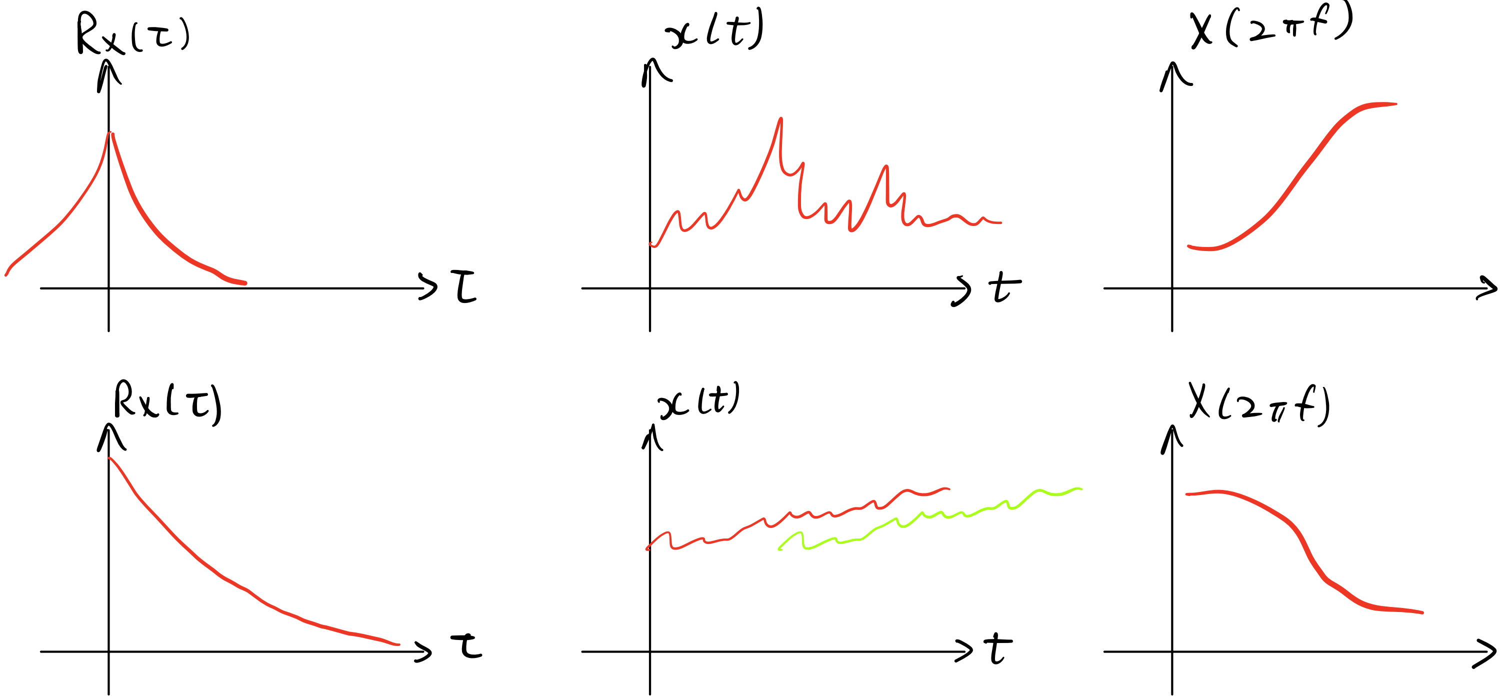
Parseval's Theorem

We can calculate the total energy in either time or frequency domain.

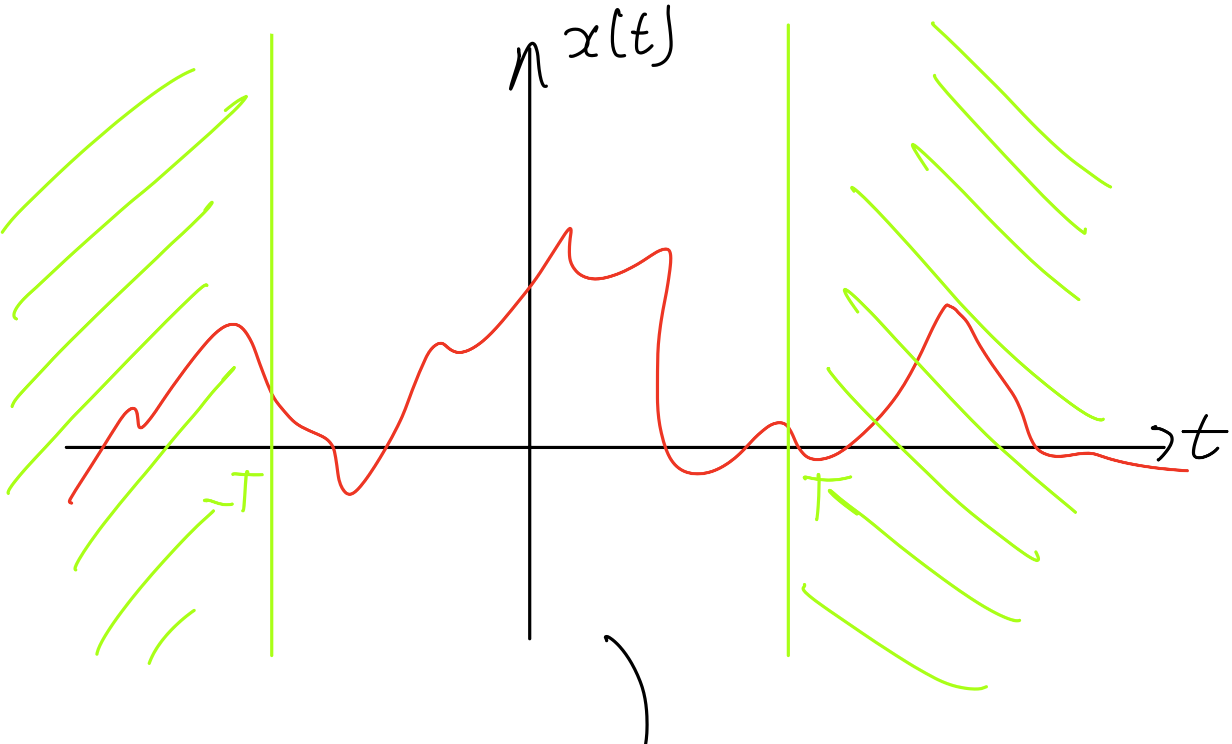


$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(2\pi f)|^2 \, df$$

Energy in time domain Energy in frequency domain.

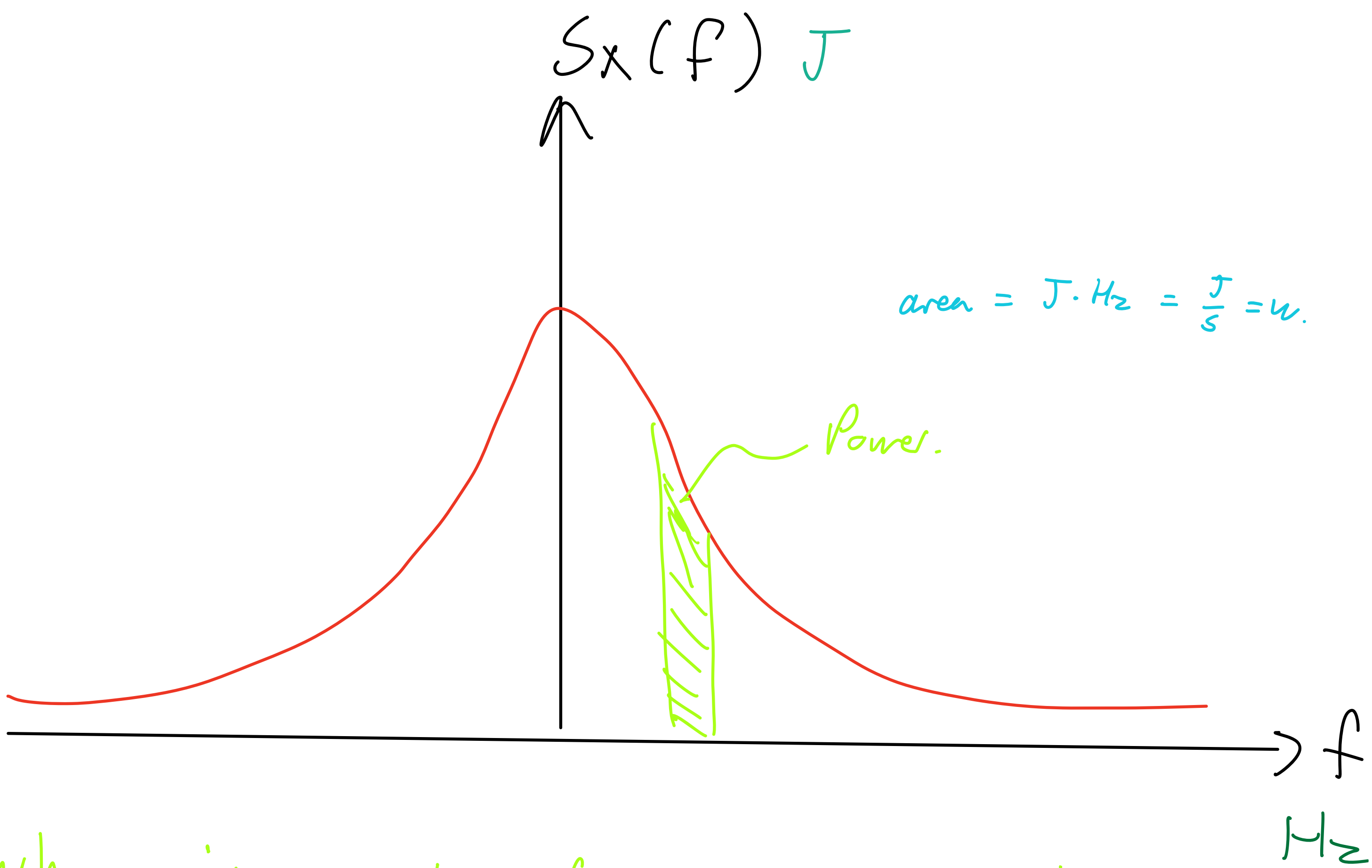


Truncating



$\rightarrow x_T(t)$

$$S_X(f) = E\Big[\underbrace{\lim_{T\to\infty}}_{\text{long term}} \underbrace{\frac{1}{2T}\left|\int_{-T}^{T} x(t)\, e^{-j2\pi f t}\, dt\right|^2}_{\text{time average.}}\Big]$$



where is most of my power located.

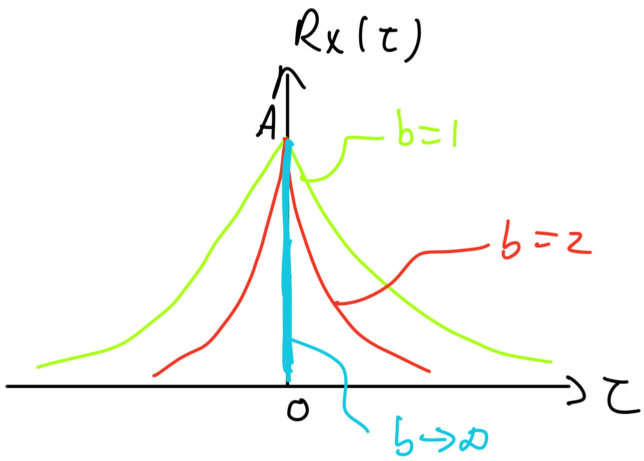
Example 15

$$R_x(\tau) = A e^{-b|\tau|} \qquad b > 0$$

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f t}\, dt = F\{R_x(\tau)\}$$

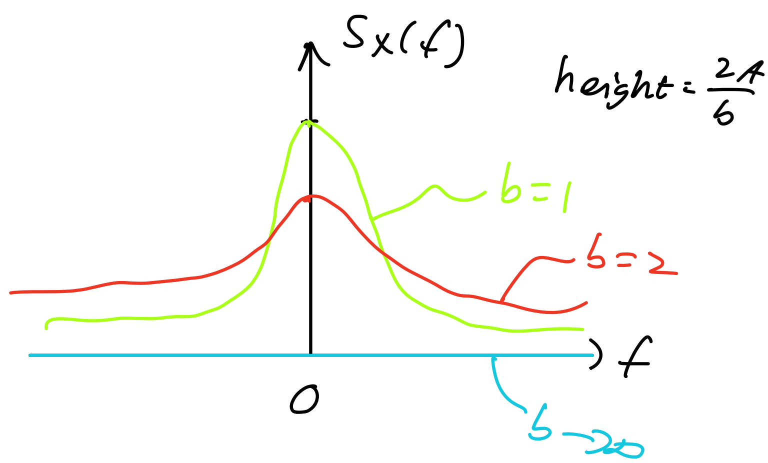
$$\underbrace{\frac{A}{b}}_{\text{Const}} \underbrace{\left[b e^{-b|\tau|}\right]}_{\text{in table.}} \xrightarrow{FT} \frac{A}{\cancel{b}}\left[\frac{2b^{\cancel{2}}}{b^2+(2\pi f)^2}\right]$$

$$= \frac{2Ab}{b^2+(2\pi f)^2}$$



$A\,\delta[\tau]$

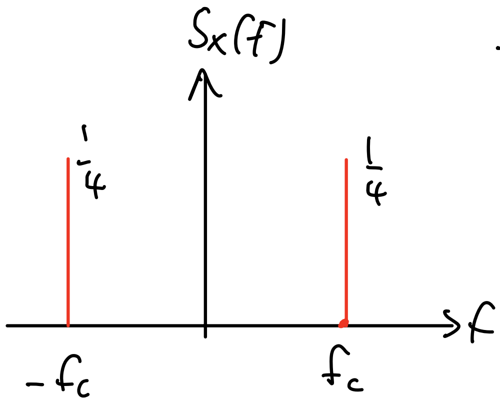
kroniker delta.



Ex 2.

$$S_X(f) = \mathcal{F}\{R_X(\tau)\} = \mathcal{F}\left\{\frac{1}{2}\cos(2\pi f_c \tau)\right\}$$

$$= \underbrace{\frac{1}{2}\cdot\frac{1}{2}}_{\frac{1}{4}}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$



$$\frac{1}{4}+\frac{1}{4} = \frac{1}{2} = P_X$$

$$R_X(\tau=0) = \frac{1}{2}\cdot 1 = \frac{1}{2}$$