

Cross Correlation and Gaussian Processes

We've looked at autocorrelation:

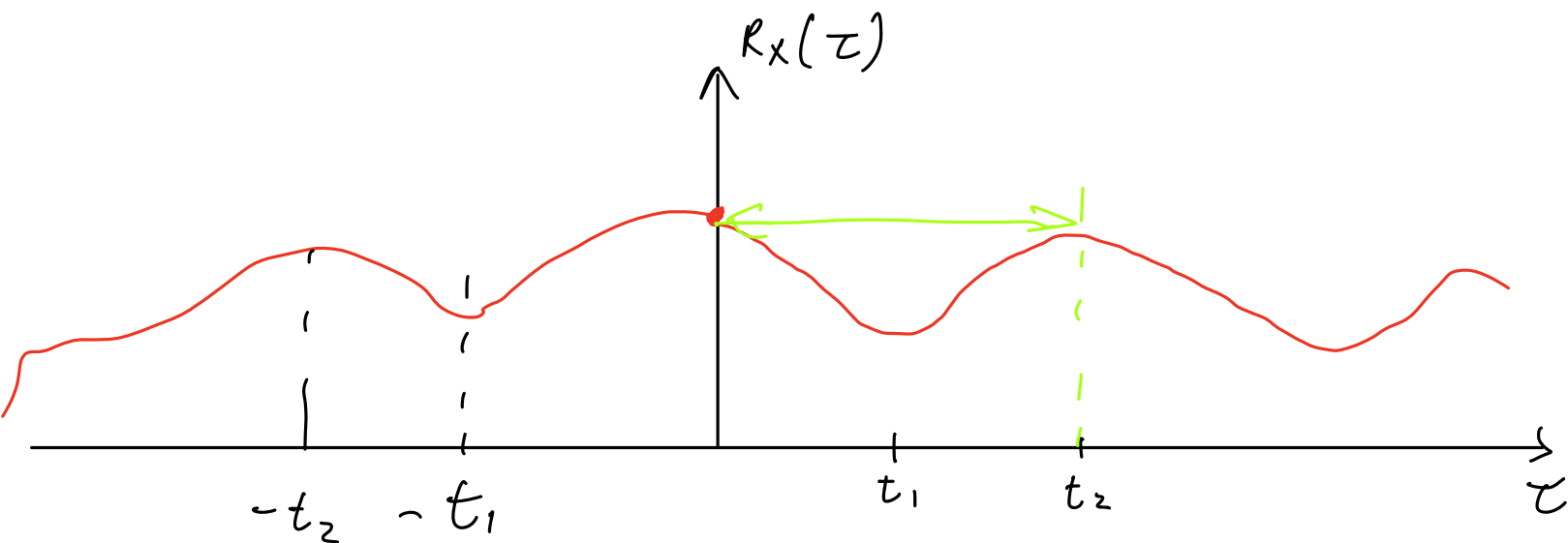
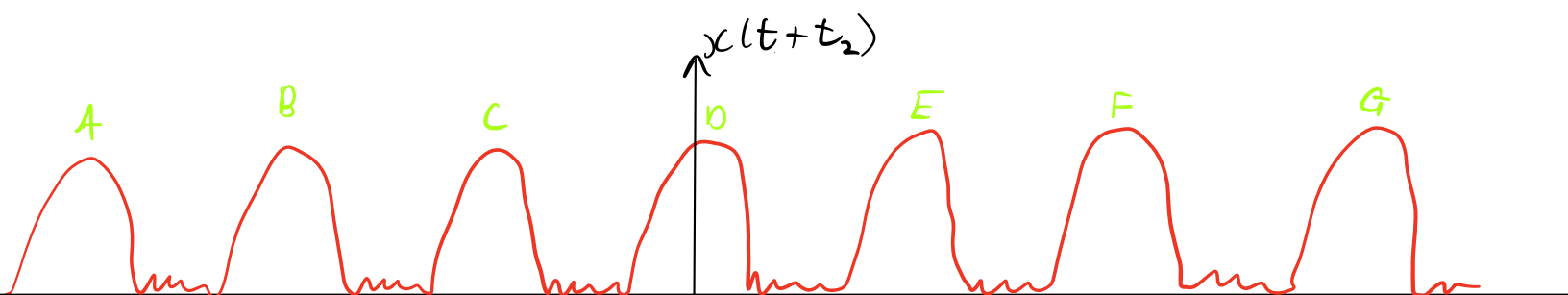
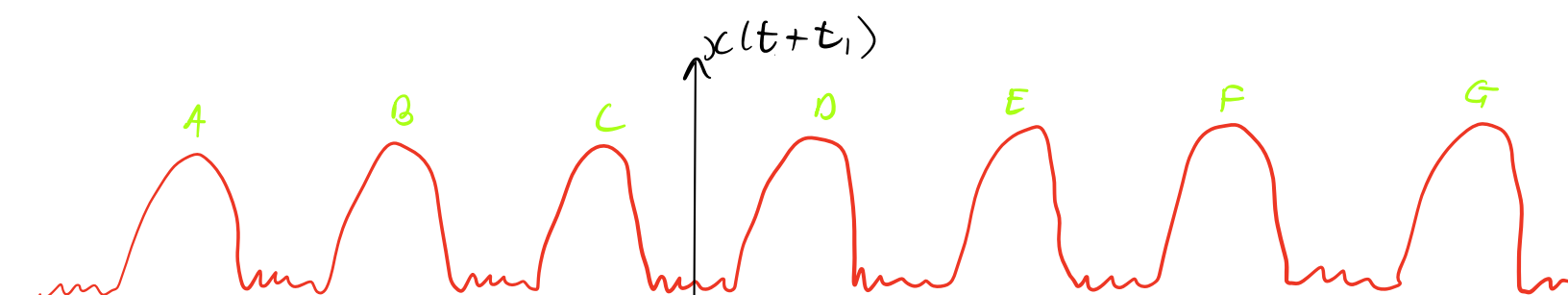
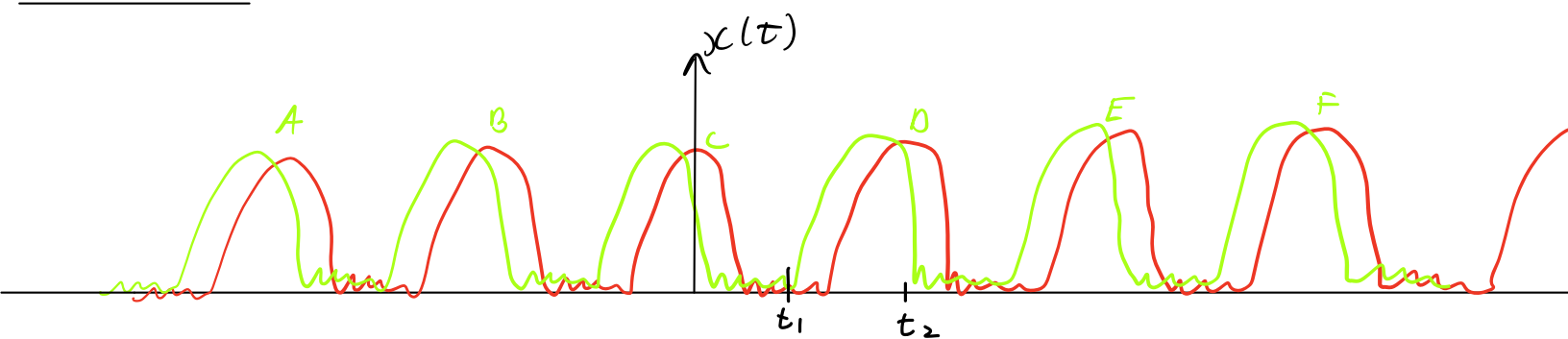
$$R_X(t, \tau) = E[X(t)X(t+\tau)]$$

Now cross-correlation:

$$R_{X,Y}(t, \tau) = E[X(t)Y(t+\tau)]$$

$$E[X^2(t)]$$

Example 1



JWSS

$X(t)$ WSS $\left\{ \begin{array}{l} \mu_x \\ R_x \end{array} \right\}$ time invariant.

$Y(t)$ WSS $\left\{ \begin{array}{l} \mu_y \\ R_y \end{array} \right\}$ time invariant.

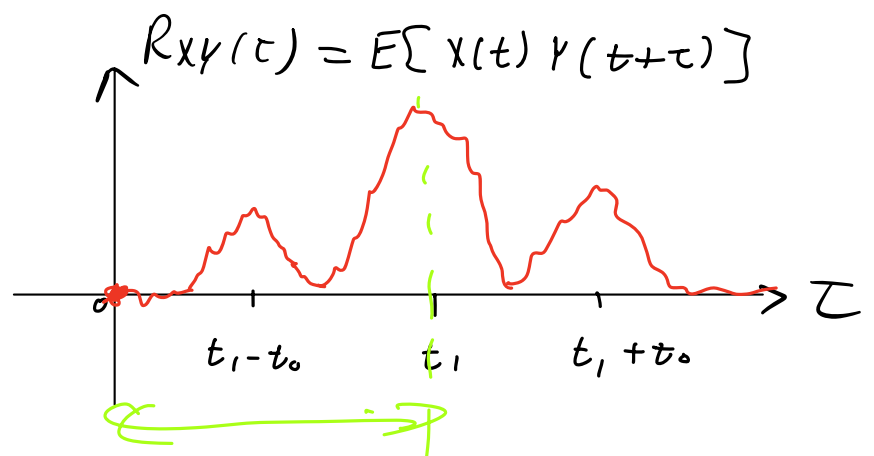
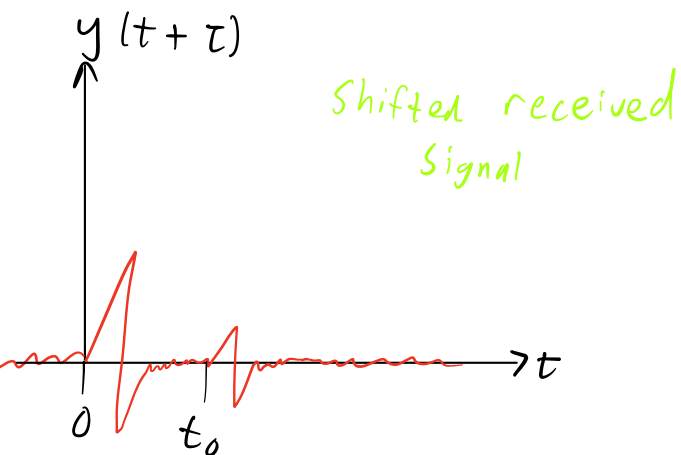
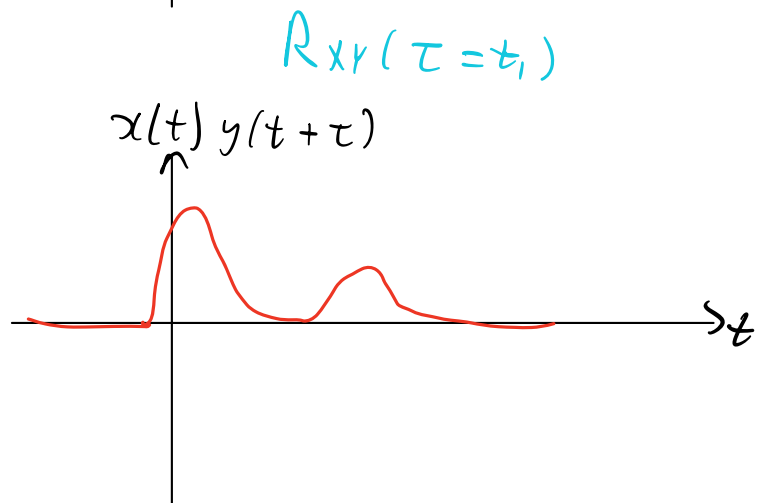
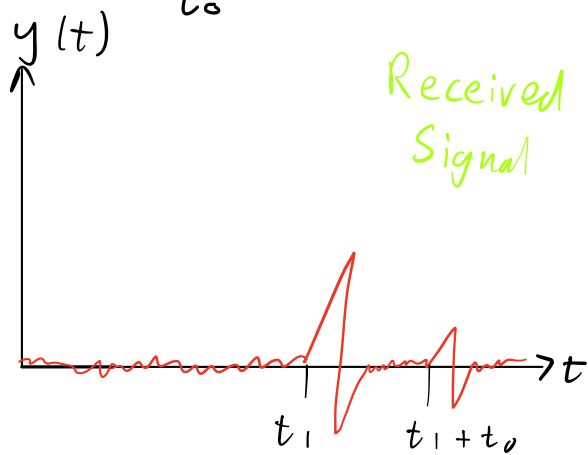
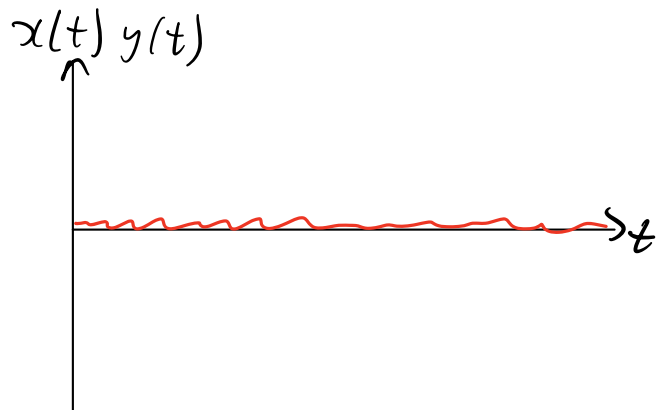
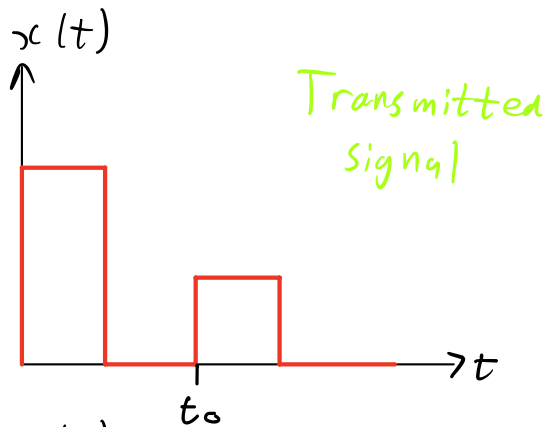
R_{xy} time invariant

τ not t .

Example 2: Radar / Sonar assuming WSS

$$R_{xy}(t, \tau) = E[X(t) Y(t+\tau)] = R_{xy}(\tau)$$

$$R_{xy}(0)$$



time $\rightarrow$ distance.



Example 13.18

Is $Y(t)$ WSS? $\rightarrow$ are $\mu_Y(t)$ and $R_Y(t, \tau)$ time-invariant

$$E[Y(t)] = E[X(t)] + E[N(t)]$$

$$= \mu_X + 0$$

$$= \mu_X \rightarrow \text{yes, time invariant.}$$

$$R_Y(t, \tau) = E[Y(t) Y(t+\tau)]$$

$$= E[(X(t) + N(t)) (X(t+\tau) + N(t+\tau))]$$

$$= E[X(t) X(t+\tau) + X(t) N(t+\tau) + N(t) X(t+\tau)$$

$$+ N(t) N(t+\tau)]$$

$$= R_X(\tau) + R_{XN}(t, \tau) + R_{NX}(t, \tau) + R_N(\tau)$$

$$= R_X(\tau) + R_N(\tau) \rightarrow \text{time invariant.}$$

$$R_{XN}(t, \tau) = E[X(t) N(t+\tau)] = E[X(t)] E[N(t+\tau)]$$

$$= 0$$

$Y(t)$ is WSS.

$$R_{YX}(t, \tau) = E[Y(t) X(t+\tau)]$$

$$= E[(X(t) + N(t)) X(t+\tau)]$$

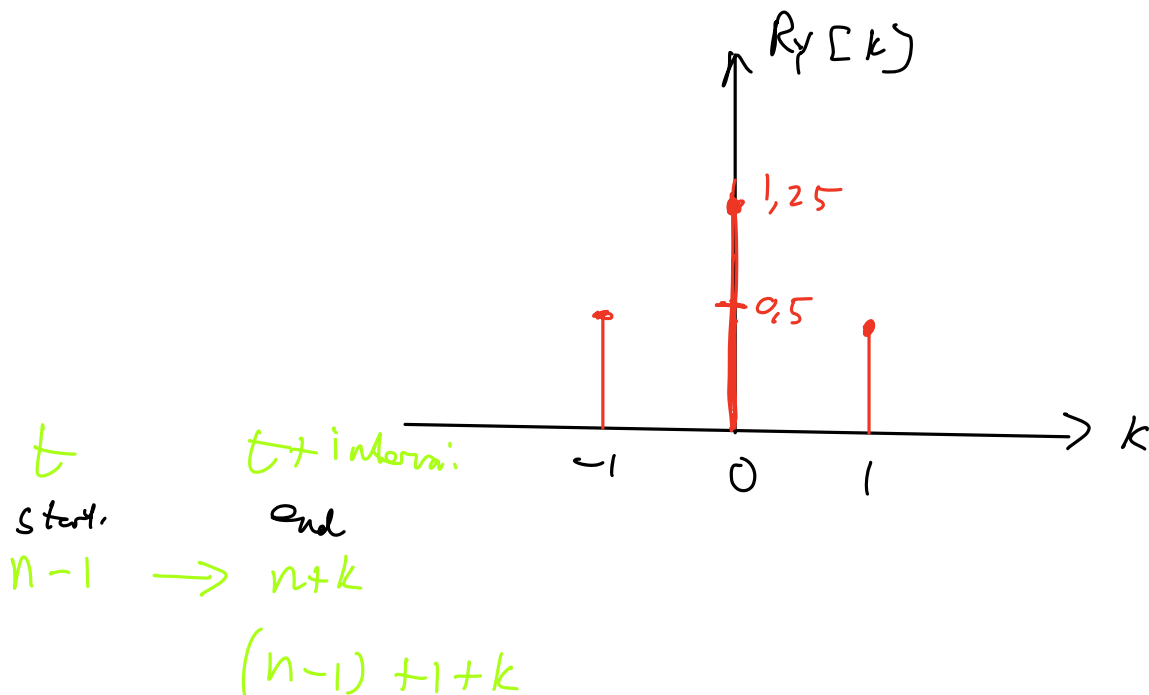
$$= E[X(t) X(t+\tau) + N(t) X(t+\tau)]$$

$$= R_X(\tau) + R_{NX}(t, \tau)$$

$Y(t)$ and $X(t)$ are JWSS.

time inv.

$$\begin{aligned}
 R_Y[k] &= E[Y_n \cdot Y_{n+k}] \\
 &= E[(X_n + 0,5 X_{n-1}) \cdot (X_{n+k} + 0,5 X_{n-1+k})] \\
 &= E[X_n X_{n+k} + 0,5 X_{n-1} X_{n+k} + 0,5 X_n X_{n-1+k} \\
 &\quad + 0,25 X_{n-1} X_{n-1+k}] \\
 &= R_X[n, k] + 0,5 R_X[n-1, k+1] + 0,5 R_X[n, k-1] \\
 &\quad + 0,25 R_X[n-1, k] \\
 &= R_X[k] + 0,5 R_X[k+1] + 0,5 R_X[k-1] + 0,25 R_X[k] \\
 &= 1,25 \delta[k] + 0,5 \delta[k+1] + 0,5 \delta[k-1]
 \end{aligned}$$



Interval =

$$\text{End} - \text{Start} = n + k - (n - 1) \\ = (k + 1)$$