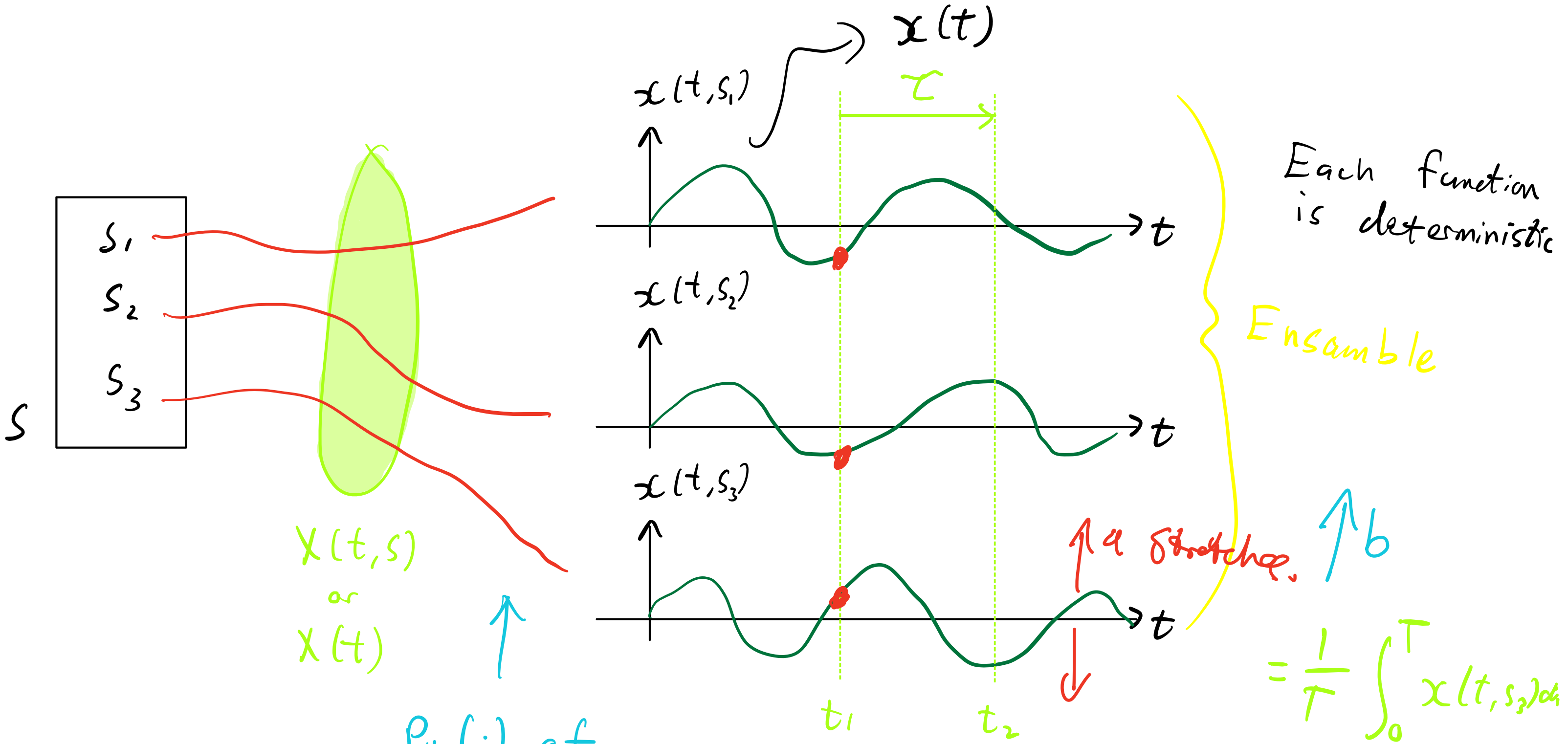
Stationary Processes



$$= \frac{1}{T}\int_0^T x(t,s_3)dt$$

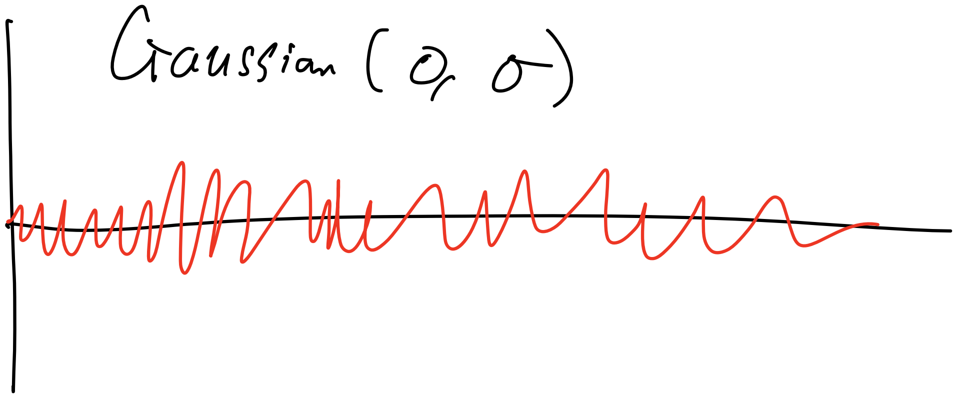
$P_X(\cdot)$ of seeing realizations.

$X(t_1)$ is just a random variable that maps $S \to$ Value.

$E[X(t_1)]$ RV. $E[X(t)]$

$f_{X(t)}(x)$

Gaussian$(0, \sigma)$

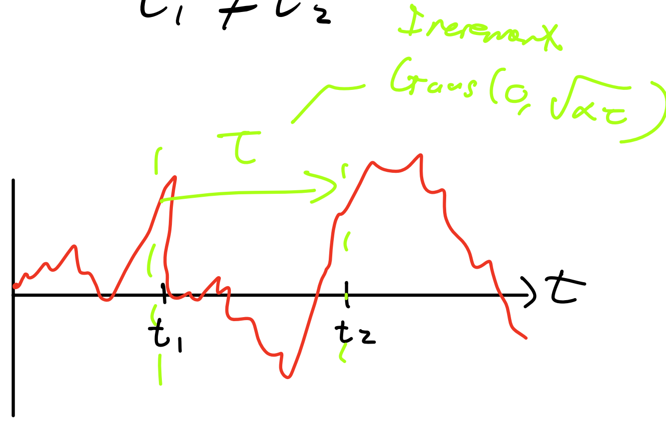


Is the Brownian Motion Process Stationary?

$W(t_1)$ is Gaussian$(0, \sqrt{\alpha t_1})$

$W(t_2)$ is Gaussian$(0, \sqrt{\alpha t_2})$

$t_1 \neq t_2$



since $t_1 \neq t_2$

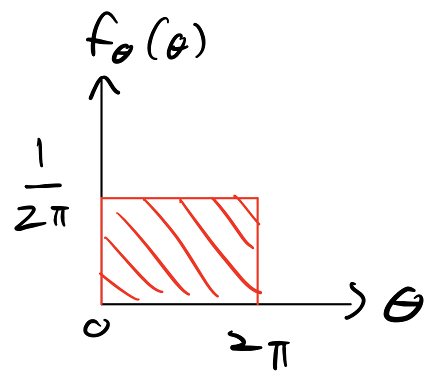
$f_{W(t_1)}(w) \neq f_{W(t_2)}(w)$

Since: $Var[W(t_1)] \neq Var[W(t_2)]$

$\therefore$ <u>not</u> stationary

Example 13.16

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi} & 0 \le \theta \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$



$\mu_X(t) = ?$ $\qquad R_X(t, \tau) = ?$

t: some t; τ: $t+\tau$ — Constant when considering phase θ

$$X(t) = \underbrace{A}_{\substack{\text{amplitude} \\ \text{(constant)}}} \cos(\underbrace{2\pi f_c t}_{\substack{\alpha \\ \text{for fixed} \\ t \text{ value}}} + \underbrace{\theta}_{RV})$$

— Function of RV θ

RV — Could also be $k\theta$ for $k \in \mathbb{Z}$

$$\mu_X(t) = E[X(t)] = E[A\cos(\alpha + k\underbrace{\theta}_{RV})]$$

$E[X] = \int x \cdot f_X(x)\, dx$

$$= A\int_0^{2\pi} \cos(\alpha + k\theta) \cdot \underbrace{\frac{1}{2\pi}}_{f_\Theta(\theta)} d\theta$$

$$= \frac{A}{2\pi}\left[\frac{\sin(\alpha + k\theta)}{k}\Bigg|_0^{2\pi}\right]$$

$$= \frac{A}{2\pi}\left[\frac{\sin(\alpha + 2\pi k) - \sin\alpha}{k}\right]$$

$$= 0$$ → time invariant

$$R_X(t,\tau) = E[X(t)\, X(t+\tau)]$$

$$= E[A\cos(2\pi f_c t + \theta) \cdot A\cos(2\pi f_c(t+\tau) + \theta)]$$

$$= \frac{A^2}{2} E[\cos(2\pi f_c \tau) + \cos(2\pi f_c(2t+\tau) + 2\theta)]$$

$$= \frac{A^2}{2}\left[E[\cos(2\pi f_c \tau)] + \underbrace{E[\cos(\underbrace{2\pi f_c(2t+\tau)}_{\alpha} + \underbrace{2\theta}_{k})]}_{=0}\right]$$

$$= \frac{A^2}{2}\cos(2\pi f_c \tau)$$ ~ time invariant.

$$\cos A \cdot \cos B = \frac{\cos(A-B) + \cos(A+B)}{2}$$

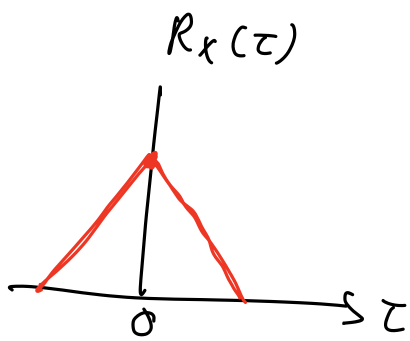
is WSS →

- $R_X(0) \geq 0$:

$$R_X(0) = R_X(t, 0) = E[X(t) \cdot X(t)]$$
$$= E[\underbrace{X^2(t)}_{\geq 0}] \geq 0 \rightarrow$$

- Symmetry:

$$R_X(\tau) = R_X(-\tau)$$

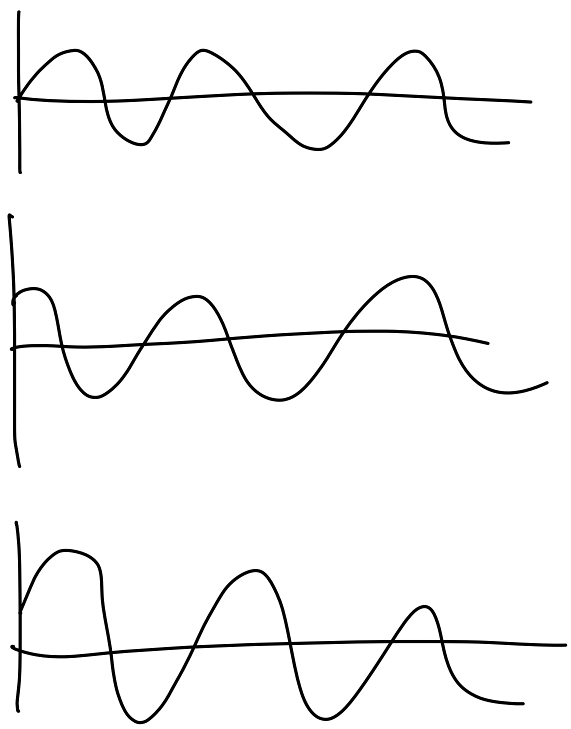


$$R_X(t, \tau) \overset{=R_X(\tau)}{=} E[X(t)\, X(t+\tau)]$$

$u = t + \tau$

$t = u - \tau$

$$= E[X(u-\tau)\, X(u)]$$
$$= E[X(u)\, X(u + (-\tau))]$$
$$= R_X(u, -\tau)$$
$$= R_X(-\tau)$$



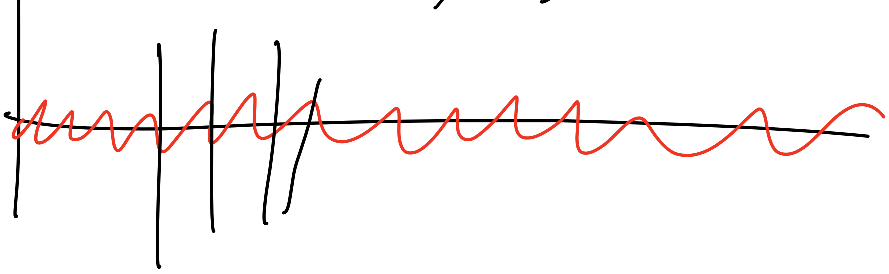
$\bar{X}(T) = 0$

$=$

$E[X(t)] = 0$

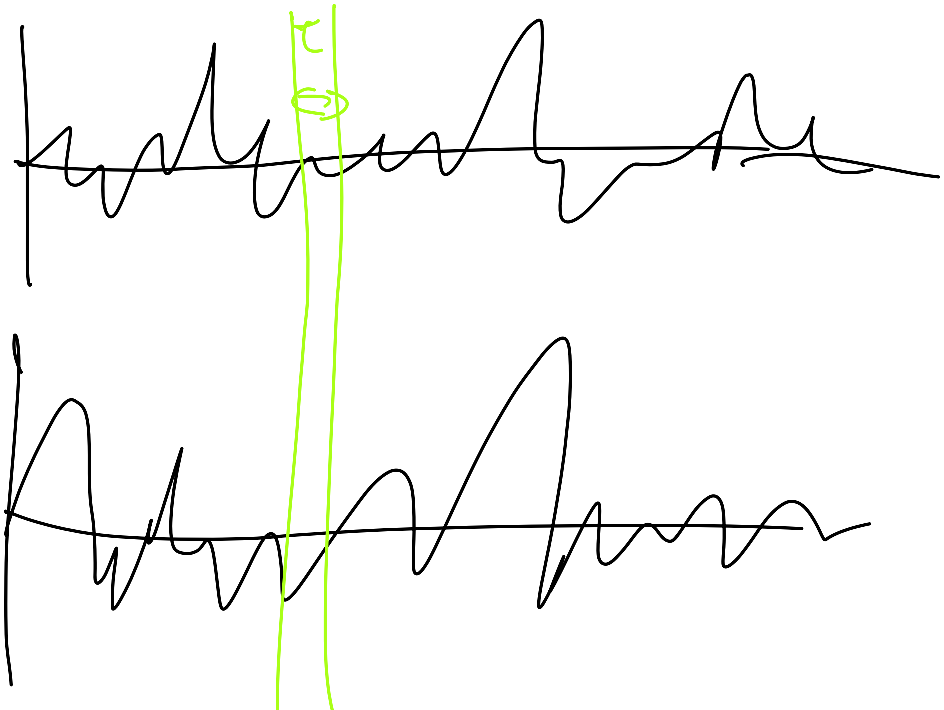
Yes ergodic.

Gausian$(0, \sigma)$



$\bar{X}(T) = 0$

$E[X(t)] = 0$ $=$ Yes ergodic.



$\bar{X}(T) = 0$

$\neq$

$E[X(t)] = \mu_x(t)$

not ergodic.