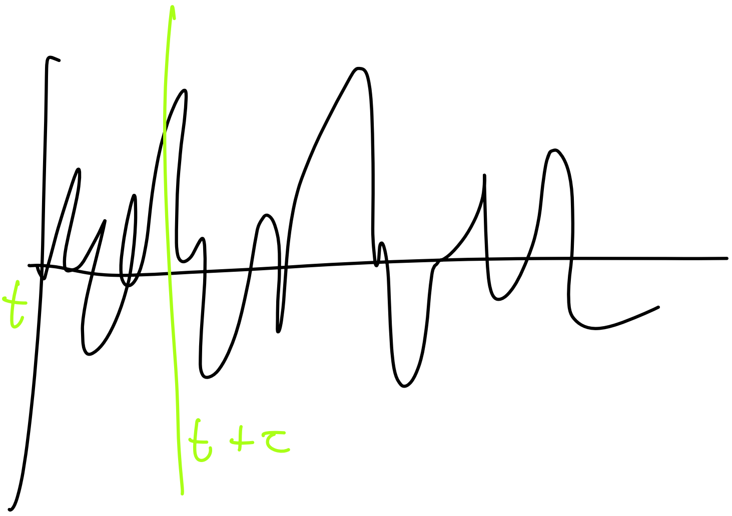
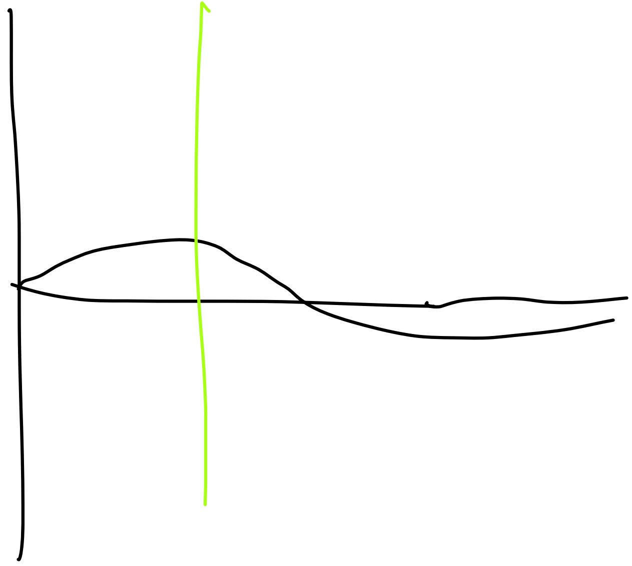
Stocastic Processes 3

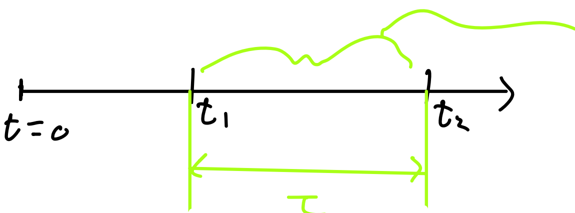


Cov → 0



Cov → large

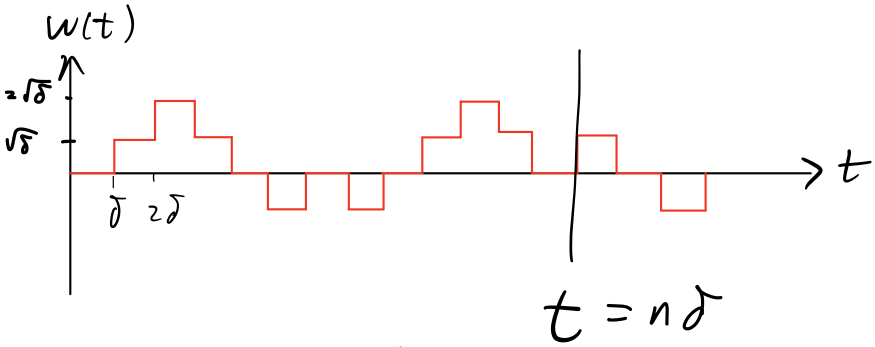
Brownian Motion Process $W(t) = \text{Gaussian}(0, \sqrt{\alpha t})$

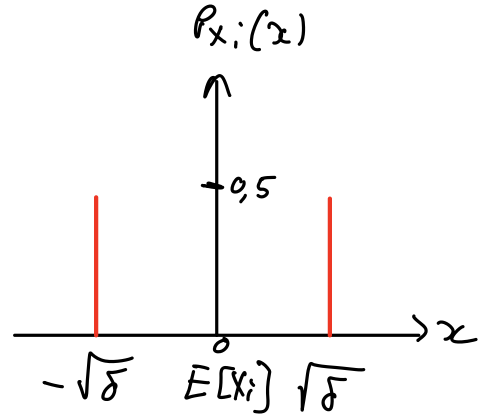


Change = $\text{Gaussian}(0, \sqrt{\alpha \tau})$

"Increment"

Symmetric Random Walk



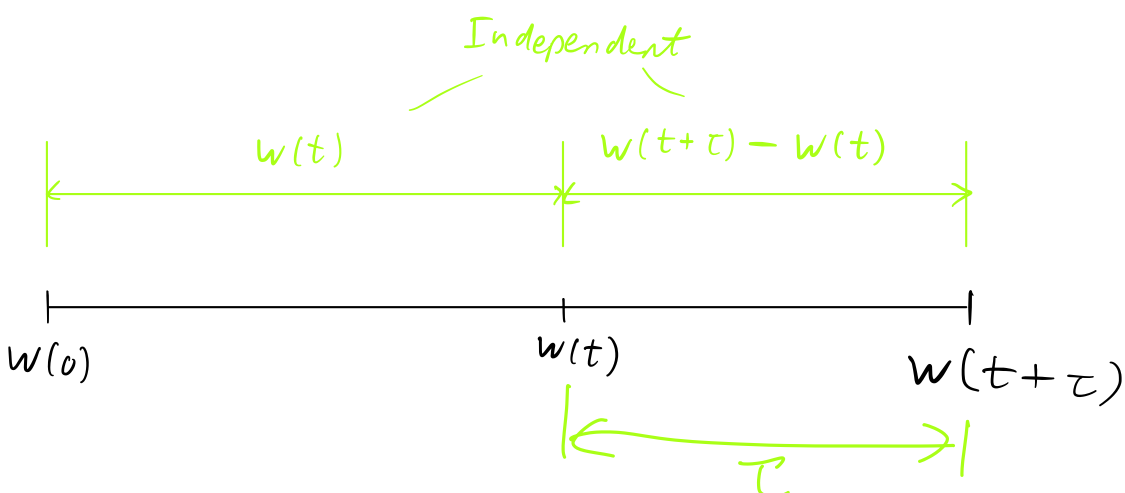


$$w(t) = W(n\delta) = \sum_{i=1}^{n} X_i$$

↳ Central limit theorem

as $n \to \infty$ $\quad w(t) = \text{Gaussian}(0, \sqrt{t})$

Brownian Motion Process

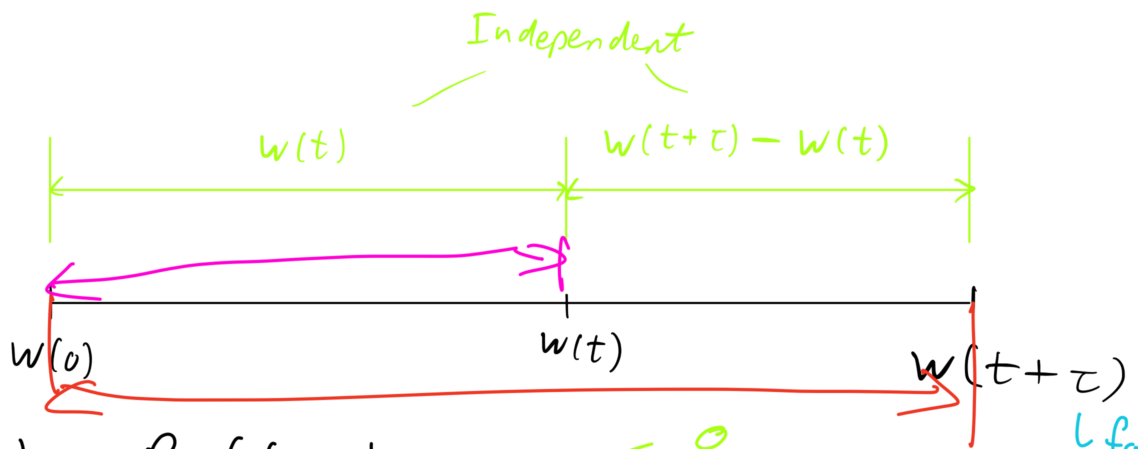


$$Cov[X, Y] = E[(X-\mu_X)(Y-\mu_Y)]$$

$$r_{XY} = E[XY]$$

Example 13.14

$$X(t) = W(t) = \text{Gaussian}(0, \sqrt{\alpha t})$$



(for $\tau > 0$

$$C_X(t,\tau) = R_X(t,\tau) - \cancel{\mu_X(t)}\,\overset{0}{\cancel{\mu_X(t+\tau)}}$$

$$= E[X(t)\,X(t+\tau)]$$

$$= E[X(t)[X(t) + [X(t+\tau) - X(t)]]]$$

$$= E[X^2(t) + X(t)[X(t+\tau) - X(t)]]$$

$$= E[X^2(t)] + E[X(t)[X(t+\tau) - X(t)]]$$

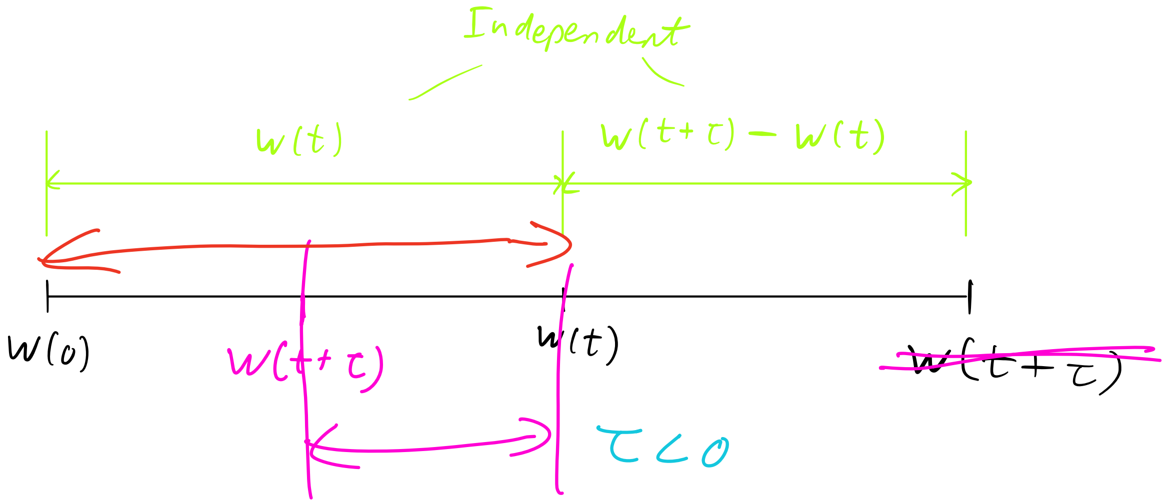
$$= E[X^2(t)] + \cancel{E[X(t)]}\,E[\overset{0}{\cancel{X(t+\tau)}} - X(t)]$$

$$= E[X^2(t)] \; \underline{-\,\mu_X^2(t)}$$

$$= \text{Var}[X(t)] = \alpha t \qquad \tau > 0$$

$$\alpha(t+\tau) \qquad \tau < 0$$

$$\boxed{C_X(t,\tau) = R_X(t,\tau) = \alpha \min(t, t+\tau)}$$



$$R_x(t,\tau) = E[X(t)X(t+\tau)]$$

$$R_x(\underset{\text{"}t\text{"}}{t+\tau}, -\tau) = E[X(t+\tau)X(t+\tau-\tau)] \qquad t+\tau \boxed{-\tau} = t$$

$$= E[X(t)X(t+\tau)]$$

$$= R_x(t,\tau)$$