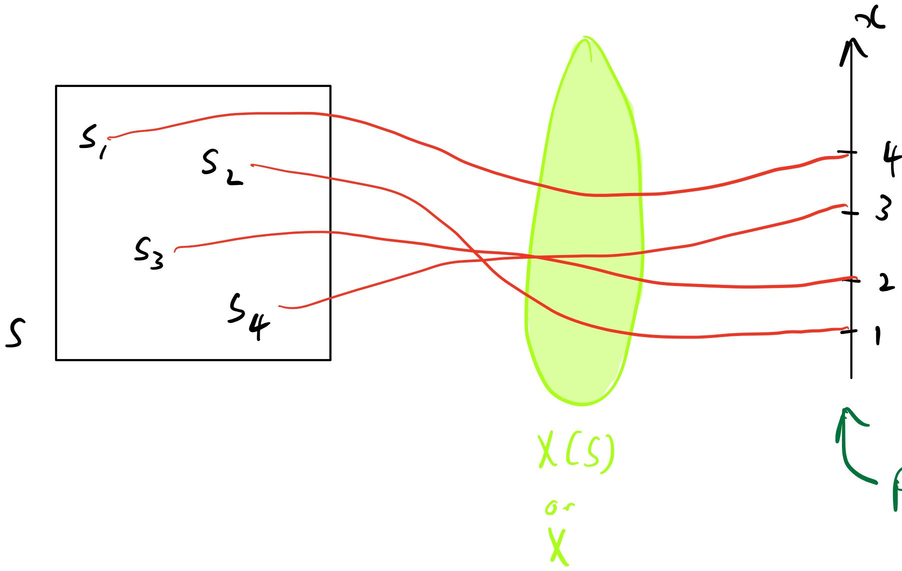
Stochastic Processes

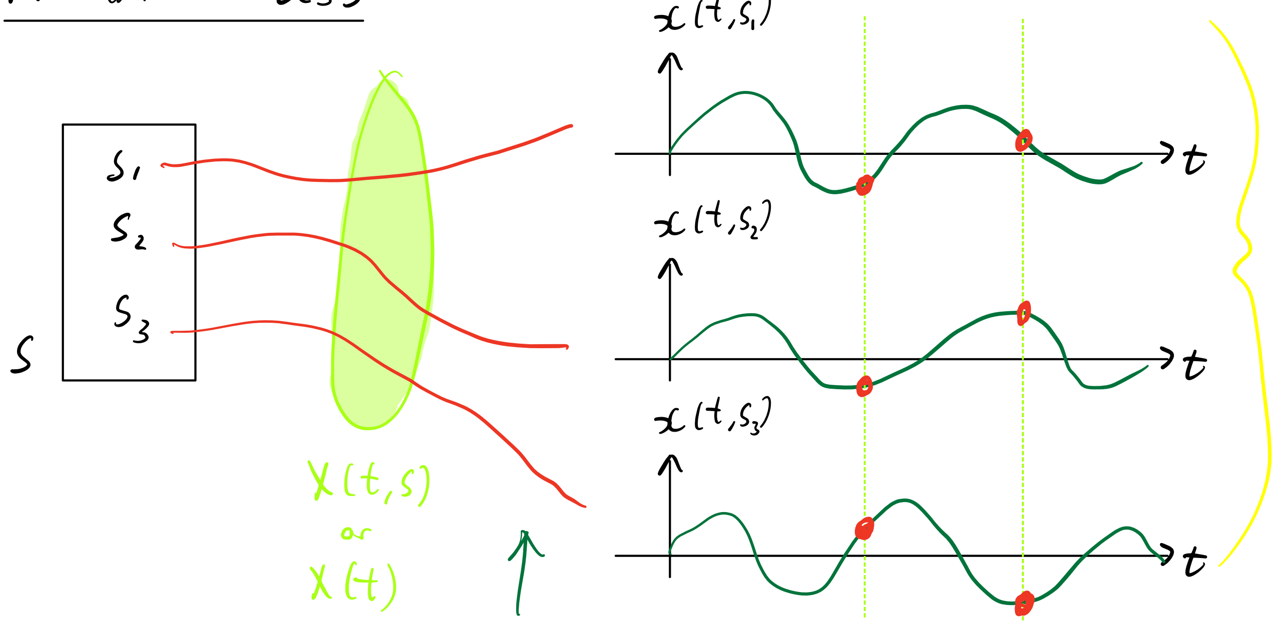
Random Variables



No concept of time or sequence of events.

$P_X(\cdot)$ of seeing outcomes.

Random Process



Each function is deterministic

$P_X(\cdot)$ of seeing realizations.

$$\bar{X} = \frac{1}{T}\int_0^T x(t,s)\, dt.$$

$X(t_1)$ is just a random variable that maps $S \to$ Value.

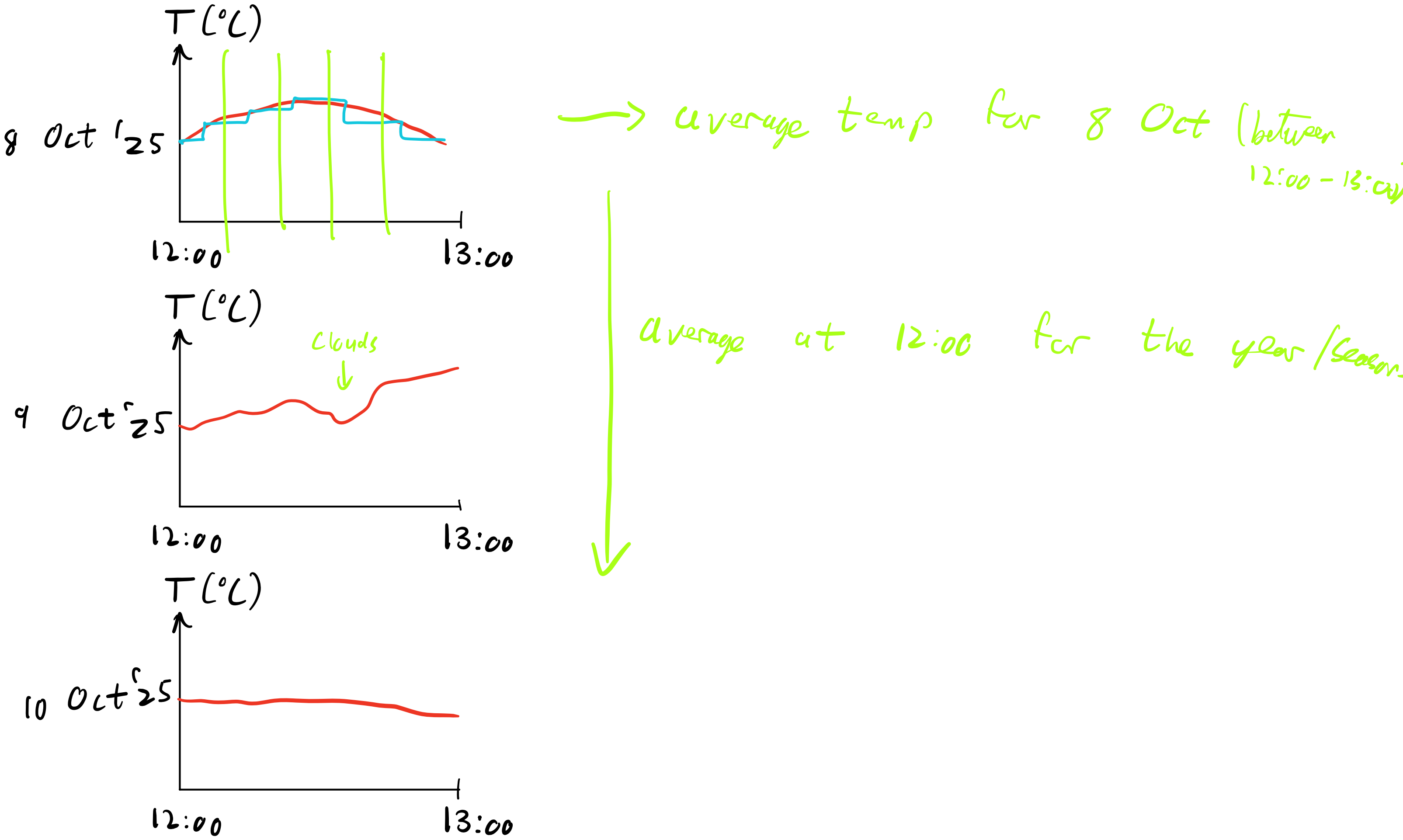
$E[X(t_1)]$

ensamble average.

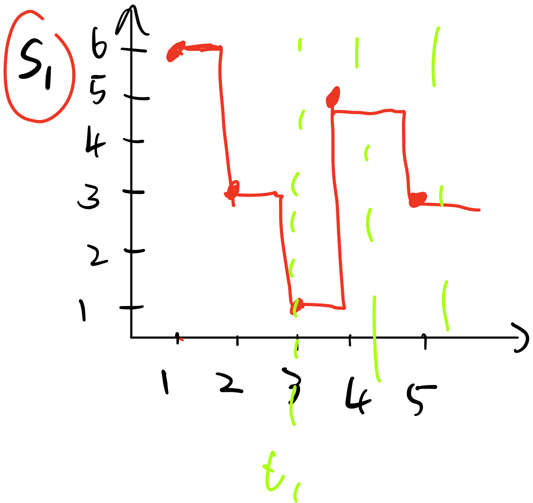
$E[X(t)] \rightsquigarrow f(t)$

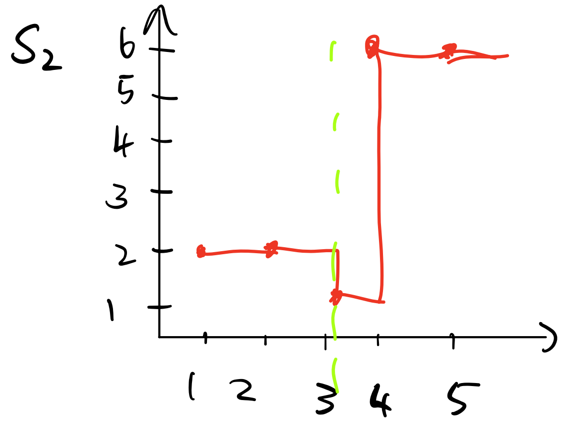
Example

The model of temperatures in Stellenbosch between 12:00 and 13:00 for every day in the year:



Example 13.5





$$x(t_1, s) = 1$$

$$X(t_1) \longrightarrow P_{X(t_1)}(x)$$

$$f_{X(t_1)}(x)$$

RV

CDF to PDF

$$F_{X(t)}(x, t) = P(X(t) \leq x)$$

$$f_{X(t)}(x,t) = \frac{d\, F_{X(t)}(x,t)}{dx}$$

$$F_{X(t_1), X(t_2)}(x_1, x_2) = P(X(t_1) \leq x_1 , X(t_2) \leq x_2)$$

Example 13.11

$$P_{X_i}(x_i) = \begin{cases} p^{x_i}(1-p)^{1-x_i} & x_i \in \{0,1\} \quad i = 1,2,\dots,n \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} p & \text{if } x_i = 1 \\ 1-p & \text{if } x_i = 0 \\ 0 & \text{otherwise} \end{cases}$$

$\boxed{\text{iid}}$

$$\bar{X} = [X_1, X_2, \dots, X_n]$$

$$P_{\bar{X}}(\bar{x}) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i}$$

$k = x_1 + x_2 + x_3 \dots + x_n$

$$= p^k (1-p)^{n-k}$$

1 0 1 1 $\leftarrow$ Random Seq.

$p = 0{,}4$

$0{,}4^{(1+0+1+1)} \cdot 0{,}6^{4-(1+0+1+1)}$

$= 0{,}0384$

$p = 0{,}5$

$0{,}5^{(1+0+1+1)} \cdot 0{,}5^{4-(1+0+1+1)}$

$= 0{,}0625$