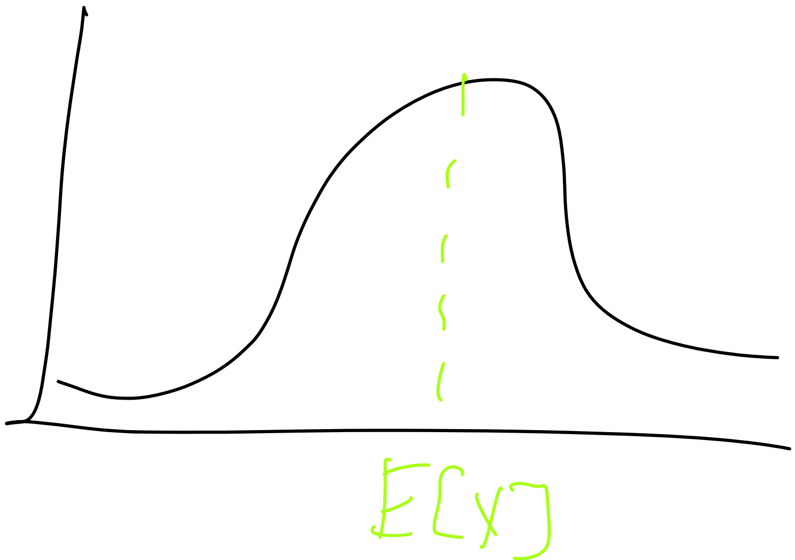
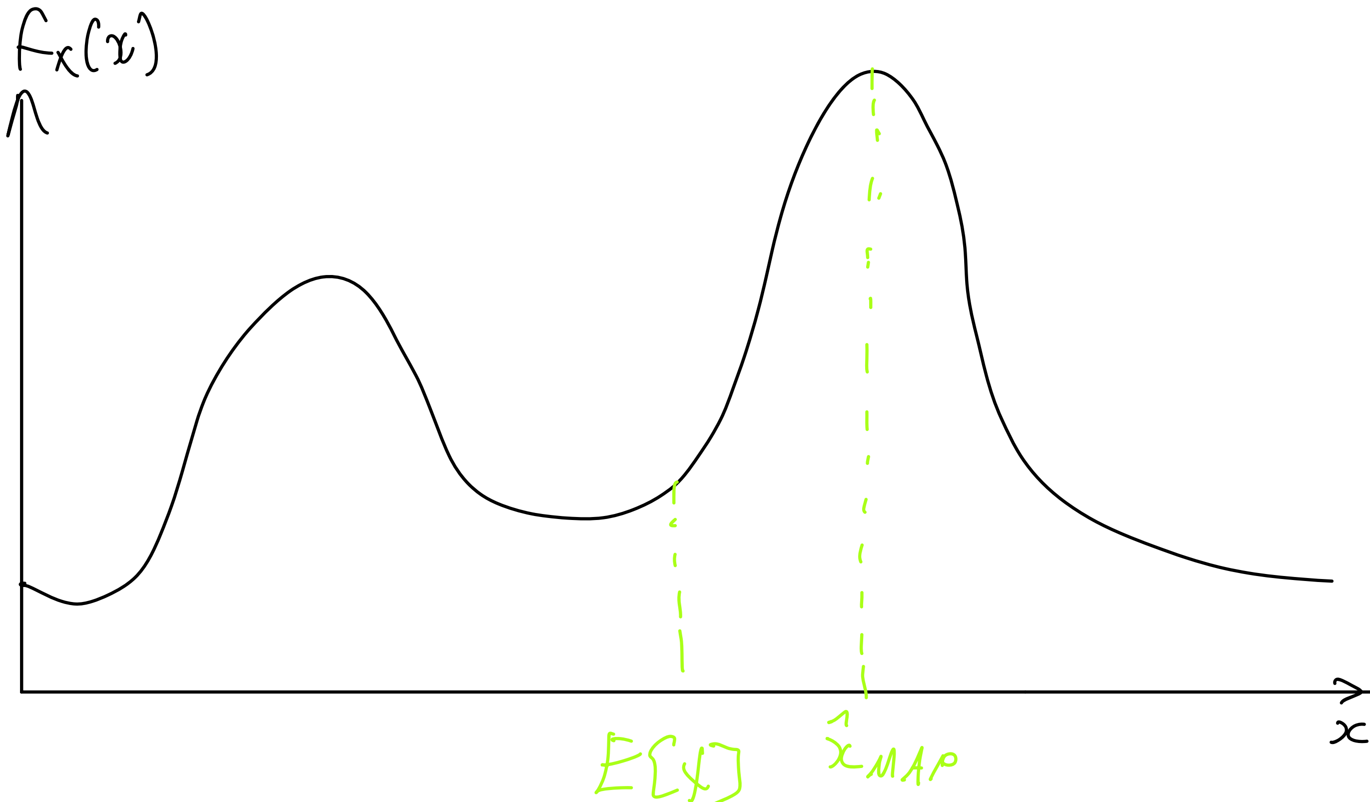
Estimation of RV





$$E[(X-\hat{X})^2]$$

Example 12.2

$$f_T(t) = \begin{cases} \frac{1}{3} e^{-\frac{t}{3}} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

mean $= E[T] = \frac{1}{\lambda}$

$= 3$ min

$\Rightarrow \lambda = \frac{1}{3}$

$A = \{T > 2\}$ given call has lasted 2 min.

how long total duration?

$$\therefore \hat{t}_A = E[T \mid T > 2] = \int_{-\infty}^{\infty} t\, f_{T \mid T>2}(t)\, dt$$

$$= \int_2^{\infty} t \cdot \frac{1}{3} e^{-\frac{t-2}{3}}\, dt$$

$= 5$ min

+3 min

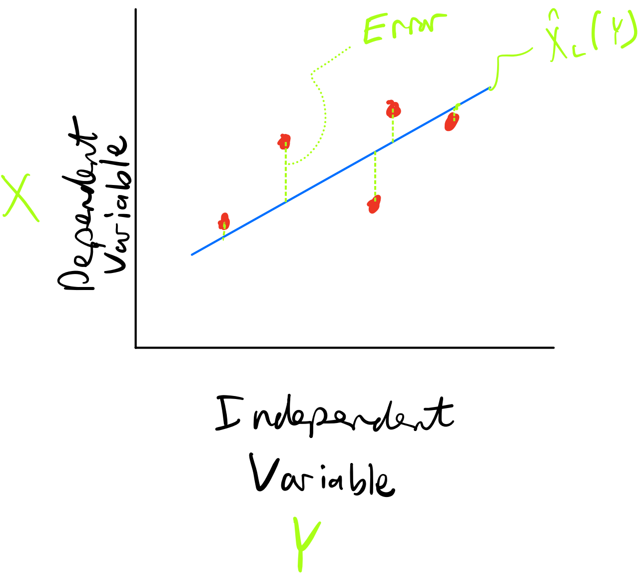
$$f_{T \mid T>2}(t) = \begin{cases} \frac{f_T(t)}{P(T>2)} & \text{for } t > 2 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \frac{1}{3} e^{-\frac{t-2}{3}} & t > 2 \\ 0 & \text{otherwise} \end{cases}$$

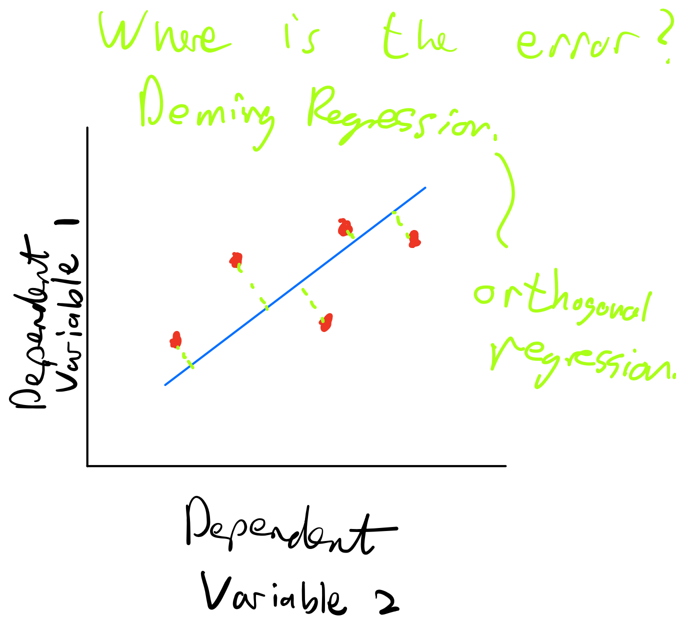
$$P(T > 2) = \int_2^{\infty} f_T(t)\, dt = 1 - F_T(2) = e^{-\frac{2}{3}}$$

$$= 1 - \int_{-\infty}^{2} f_T(t)\, dt$$

Memory-less property

LMSE





Example 12.7

a) $\hat{q}_B = E[Q] = \frac{i}{i+j} = \frac{2}{2+2} = 0{,}5$

b) $\hat{q}_{ML} = \underset{q}{\arg\max}\, P_{K|Q}(k|q)$

$= \underset{q}{\arg\max} \binom{n}{k} q^k (1-q)^{n-k}$

To get max: $\frac{d\, P_{K|Q}(k|q)}{dq} = 0$

Product rule.

$$\frac{d\left[\underbrace{q^{k}}_{v}\,\underbrace{(1-q)^{n-k}}_{u}\right]}{dq} = 0$$

$$\frac{dv}{dq}u + \frac{du}{dq}v = 0$$

$$k\,q^{k-1}\cdot(1-q)^{n-k} + \left[-(n-k)(1-q)^{n-k-1}\,q^{k}\right] = 0$$ chain rule

for $q \neq 1$: $$k\,q^{k-1} = \frac{n-k}{1-q}\,q^{k}$$

for $q \neq 0$: $$\frac{k}{q} = \frac{n-k}{1-q}$$

$$k - \cancel{kq} = nq - \cancel{kq}$$

$$q = \frac{k}{n} \Rightarrow \hat{q}_{ML} = \frac{k}{n} = \frac{\#\text{ heads}}{\#\text{ flips}}$$

$= P(\text{heads})$.

c) $\hat{q}_{MAP}(k) = \dfrac{k+1}{n+2}$ → biased away from 0 and

d) $\hat{q}_{M}(k) = \dfrac{k+2}{k+4}$

e) $\hat{q}_{L}(k) = \hat{q}_{M}(k)$

Example 12.7

a) $\hat{q}_B = E[Q] = \dfrac{i}{i+j}$ (formula sheet)

$= \dfrac{2}{2+2} = \dfrac{1}{2}$

b) $\hat{q}_{ML}(k) = \underset{q}{\arg\max}\ P_{K|Q}(k|q) = \binom{n}{k} \underbrace{q^k}_{v} \underbrace{(1-q)^{n-k}}_{u}$ (Binomial)

To get max: $\dfrac{d P_{K|Q}(k|q)}{dq} = 0$

$\Rightarrow \binom{n}{k}\left(\dfrac{dv}{dq} u + \dfrac{du}{dq} v\right) = 0$ (Product rule)

$k q^{k-1}(1-q)^{n-k} + \left[-(n-k)(1-q)^{n-k-1} q^k\right] = 0$ (chain rule)

for $q \neq 1$: $k q^{k-1} = \dfrac{(n-k)}{(1-q)} q^k$

for $q \neq 0$: $\dfrac{k}{q} = \dfrac{(n-k)}{(1-q)}$

$k - \cancel{kq} = nq - \cancel{kq}$

$q = \dfrac{k}{n} \Rightarrow \hat{q}_{ML} = \dfrac{k}{n} \Rightarrow \dfrac{\#\text{heads}}{\#\text{flips}}$

c) $\hat{q}_{MAP}(k) = \dfrac{k+1}{n+2}$