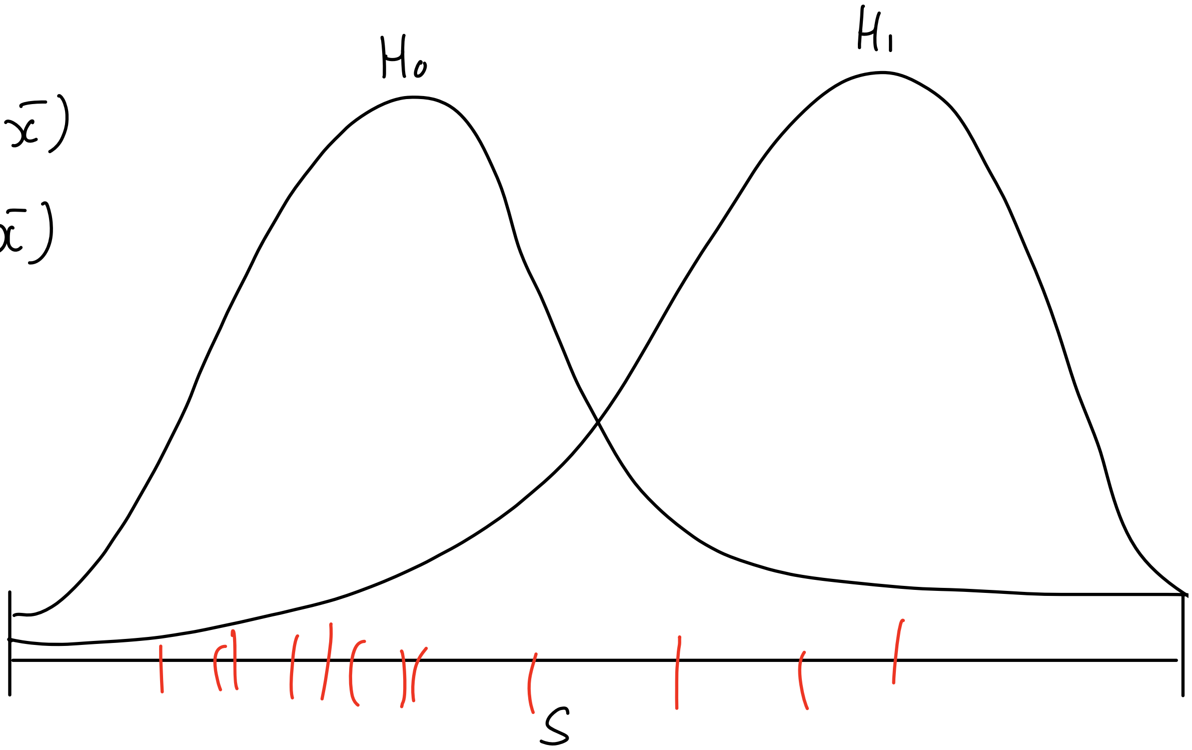
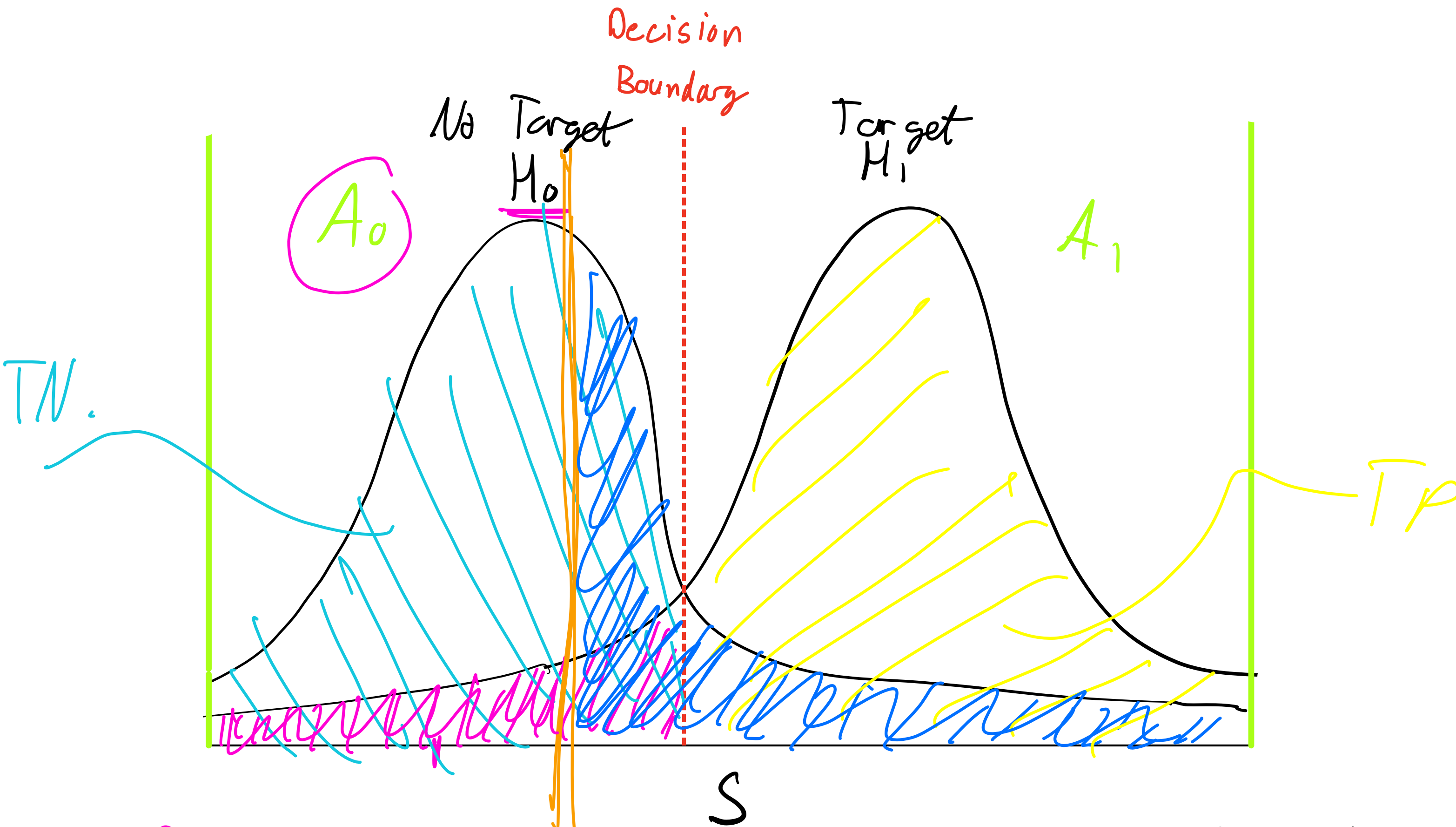
Hypothesis testing

Binary Hypothesis Testing

$P_{\bar{X}|H_0}(\bar{x})$

$P_{\bar{X}|H_1}(\bar{x})$





P_{FN} = Type II

Miss.

False Negative.

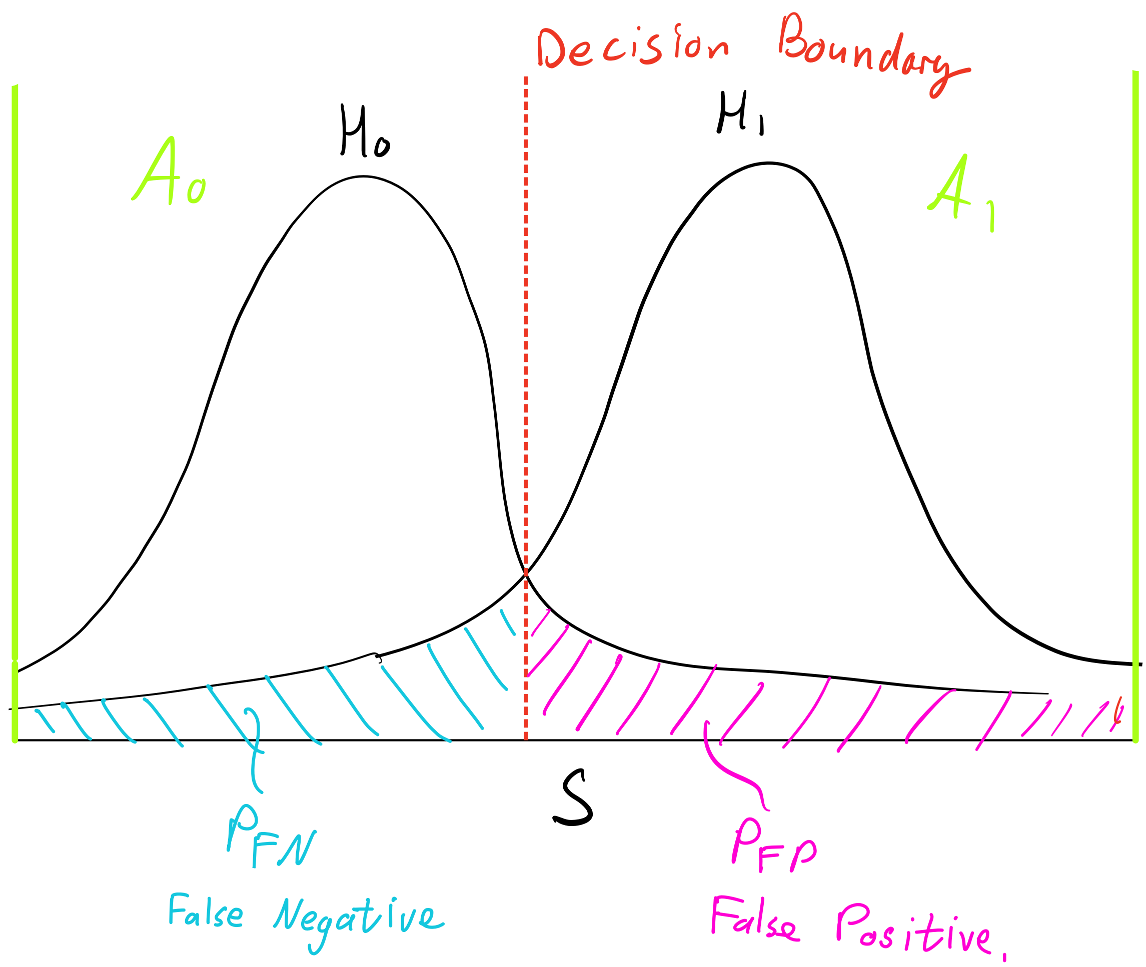
P_{FP} = Type I $= P(A_1 | H_0)$

False Alarm.

False Positive.

Radar Example

		Truth/Reality	
		Negative	Positive
		H_0 : no target	H_1 : target Present
Predict	Negative: Decide no target Present A_0	✓ (true negative)	Type II error (false negative)
	Positive: Decide target present A_1	Type I error (false positive)	✓ (true positive)

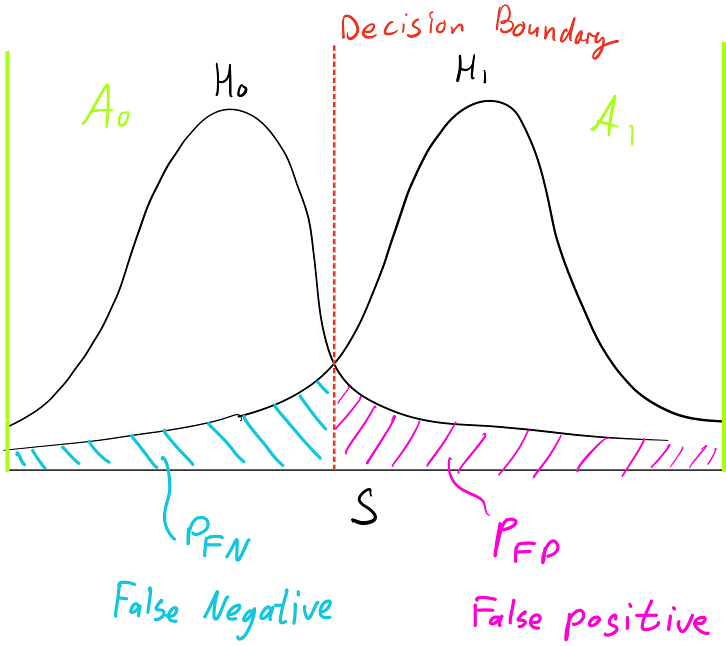


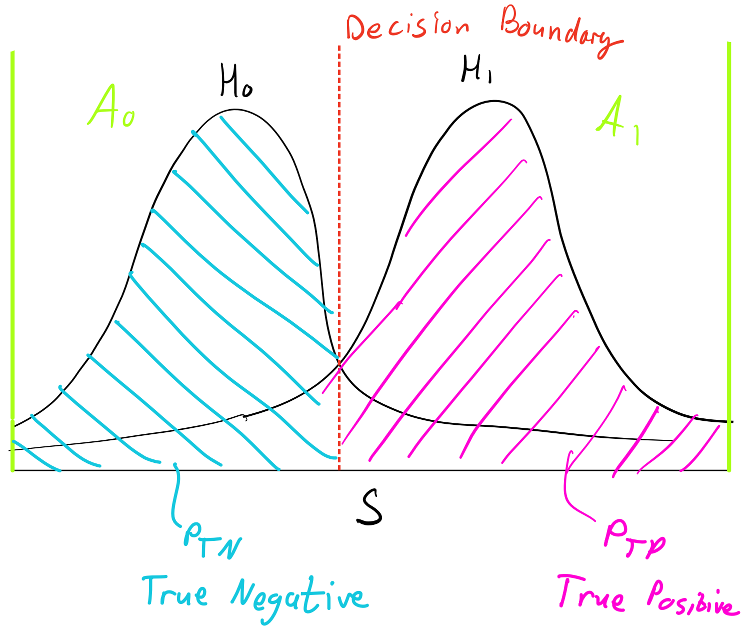
- $P_{FP} = \underbrace{P_{FR}}_{\text{false reject}} = P(A_1 \mid H_0)$ Type I

- $P_{FN} = \underbrace{P_{FA}}_{\text{false accept}} = P(A_0 \mid H_1)$ Type II

Decision Boundary balance these.:

eg. $A_0 = \phi$, $A_1 = S$ $\Rightarrow$ $P_{FN} = 0$, $P_{FP} = 1$ | $A_0 = S$, $A_1 = \phi$ $\Rightarrow$ $P_{FN} = 1$, $P_{FP} = 0$



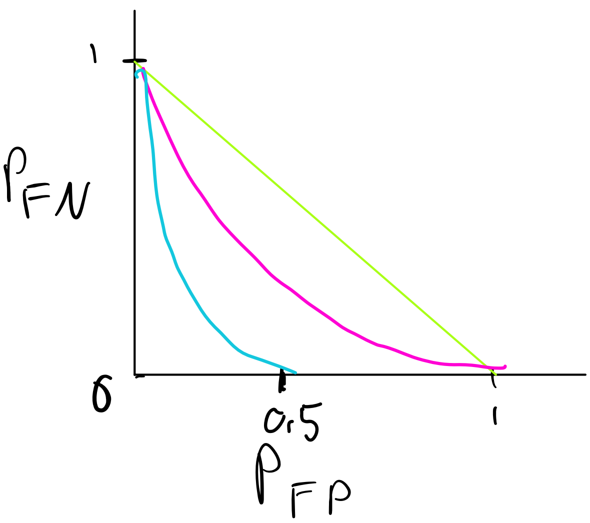


if "no target" = H_0

DET

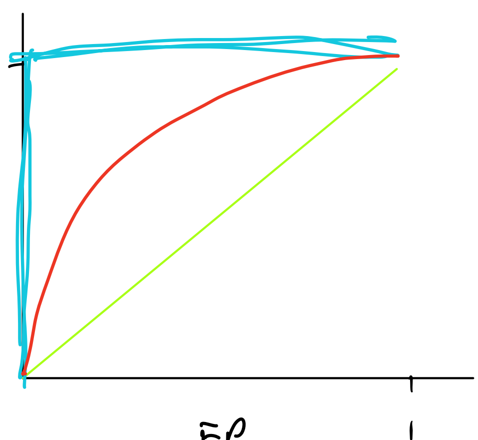
ROC

Plots "drawn" as decision boundary shifts.



(Sensitivity)

$$TPR = \frac{TP}{TP + FN}$$



$$FPR = \frac{FP}{FP + TN}$$

(1 − Specificity)

MAP $\quad P_{ERR} = P(A_1 \cap H_0) + P(A_0 \cap H_1)$

Choose $s \in A_0$ if $P(H_0 | s) \geq P(H_1 | s)$

otherwise Choose $s \in A_1$

Proof:

$s \in A_0$ if $\underbrace{P(s|H_1)P(H_1)}_{\substack{P(s \cap H_1) \\ A_0 \text{ error.}}} \leq \underbrace{P(s|H_0)P(H_0)}_{\substack{P(s \cap H_0) \\ A_0 \text{ Correct}}}$

apply Bayes: $P(H_1|s)\cancel{P(s)} \leq P(H_0|s)\cancel{P(s)}$

also note.

$$\frac{P(s|H_0)}{P(s|H_1)} \geq \frac{P(H_1)}{P(H_0)}$$

Example. 11.7

$\underbrace{q_0 = 10^{-4}}_{H_0}$ Good

$\underbrace{q_1 = 10^{-1}}_{H_1}$ Problem

Problems once every 10 days

$\therefore P(H_1) = 0{,}1$
$P(H_0) = 0{,}9$
$\Big\}$ ME

N?
take samples → choose model

$$P_{FA} = P_{FP} = P(A_1 \mid H_0)$$

$$P_{MISS} = P_{FN} = P(A_0 \mid H_1)$$

$$P_{ERR} = P(A_1 \mid H_0)\,P(H_0) + P(A_0 \mid H_1)\,P(H_1)$$

N is Geometric (# tests until failure)

$$\Rightarrow E[N] = \frac{1}{q_i}$$

$$P_{N \mid H_i}(n) = \begin{cases} q_i(1-q_i)^{n-1} & n = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

MAP test:

$n \in A_0$ if $\dfrac{p_{N|H_0}(n)}{p_{N|H_1}(n)} \geq \dfrac{P(H_1)}{P(H_0)}$ $n \in A_1$ otherwise

$$\frac{q_0(1-q_0)^{n-1}}{q_1(1-q_1)^{n-1}} \geq \frac{0,1}{0,9}$$

$\vdots$

$$n \geq 1 + \frac{\ln\left(\frac{q_1}{q_0} \cdot \frac{1}{9}\right)}{\ln\left(\frac{1-q_0}{1-q_1}\right)} = 45,8$$

$\therefore A_0 = \{n \geq 46\} \Rightarrow A_1 = \{n \leq 45\}$

$\Rightarrow$ if failure before 46 then reject H_0 (accept H_1)

if 45 pass then fail to reject H_0 (accept H_0)

false alarm

$$P_{FA} = P(A_1 | H_0) = P(\underbrace{N \leq 45}_{\text{fail in first 45}} | H_0) = F_{N|H_0}(45) = 0,0045$$

$$P_{MISS} = P(A_0 | H_1) = P(N > 45 | H_1) = 1 - F_{N|H_1}(45) = 0,0087$$

$$\Rightarrow P_{ERR} = P(A_1|H_0)P(H_0) + P(A_0|H_1)P(H_1)$$

$$= 0,0049 \approx 0,5\%$$