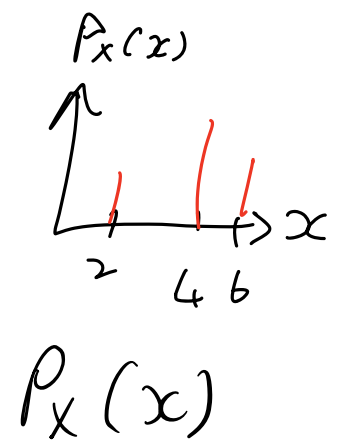
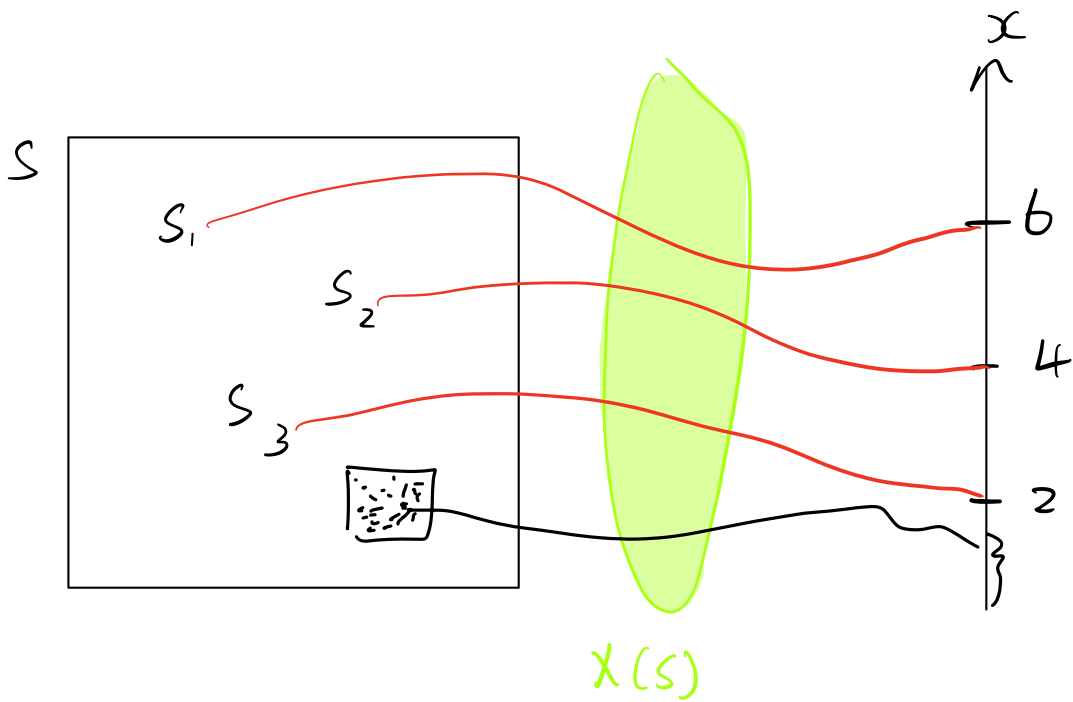
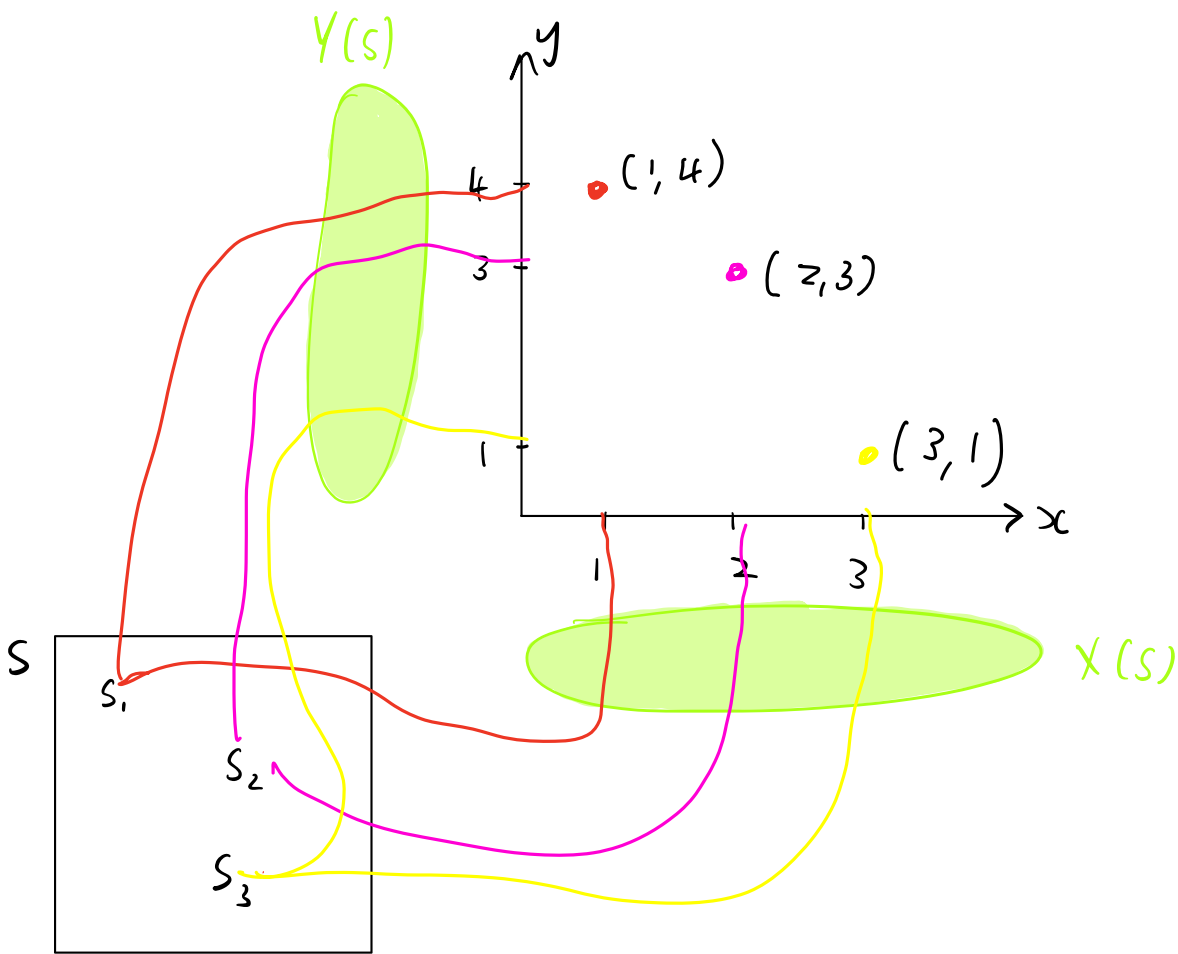
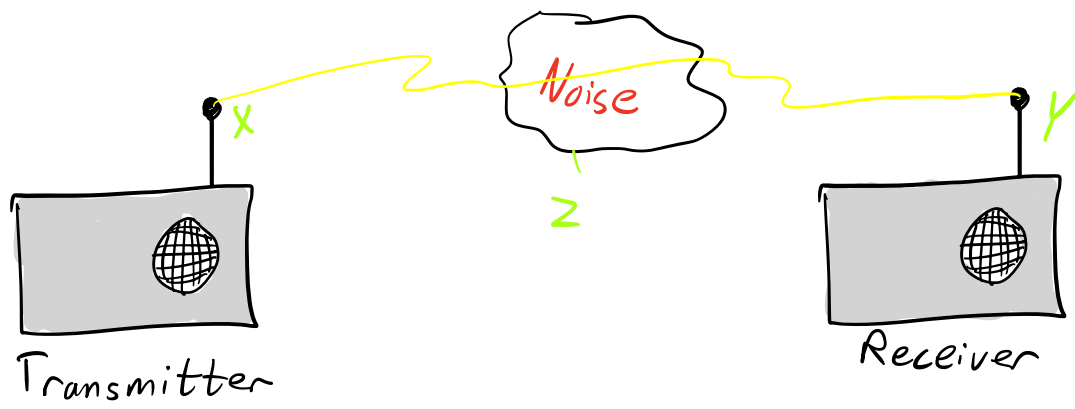






$$\begin{bmatrix} X & Y \end{bmatrix}' = \begin{bmatrix} X \\ Y \end{bmatrix} \quad \bar{X} = \cancel{X} = [x_1 \ x_2 \ \dots \ x_n]'$$

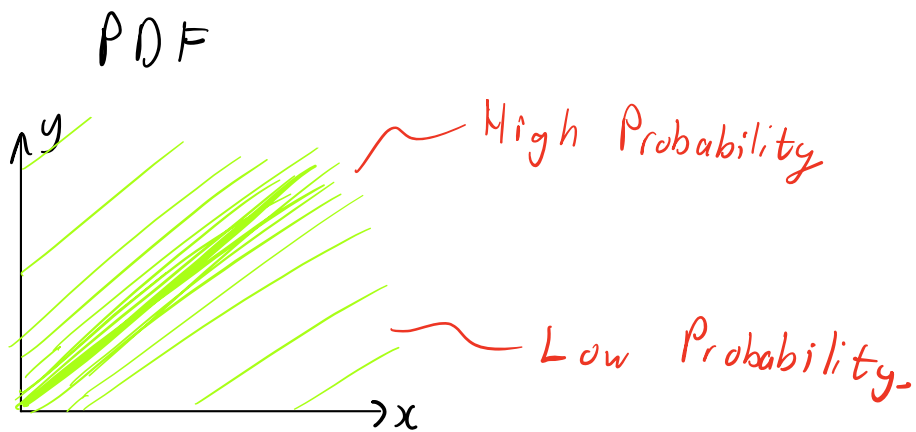


$$Y = X + Z$$

(X, Y)

(X, Z)

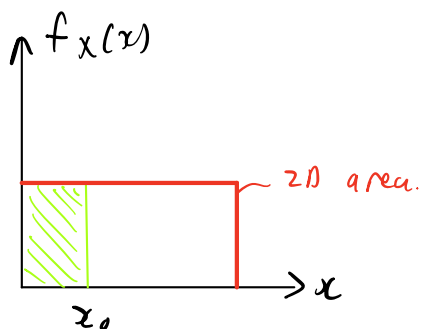
(Y, Z)



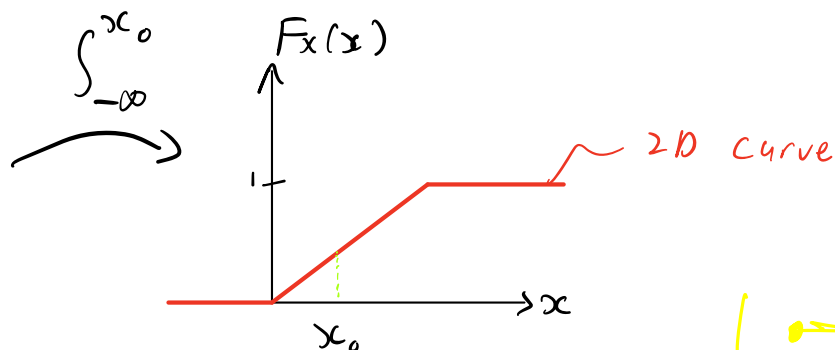
CDF (X, Y means $X \cap Y$ or " X and Y ")

1 Random Variable $F_X(x_0) = P(X \leq x_0)$

PDF

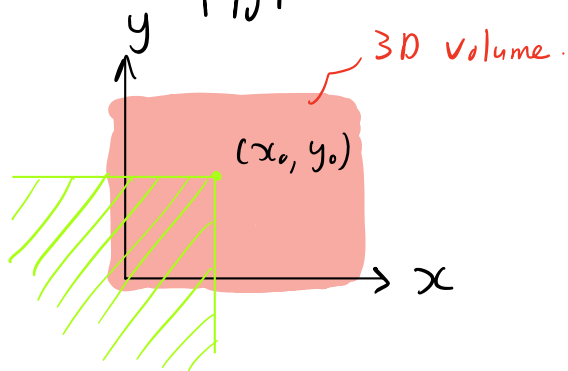


CDF



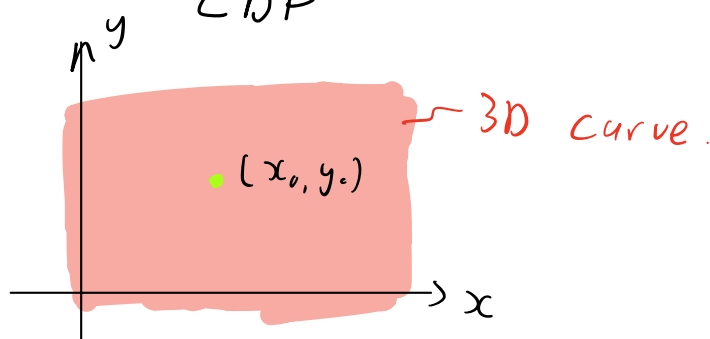
2 Random Variables $F_{X,Y}(x_0, y_0) = P(X \leq x_0, Y \leq y_0)$

PDF



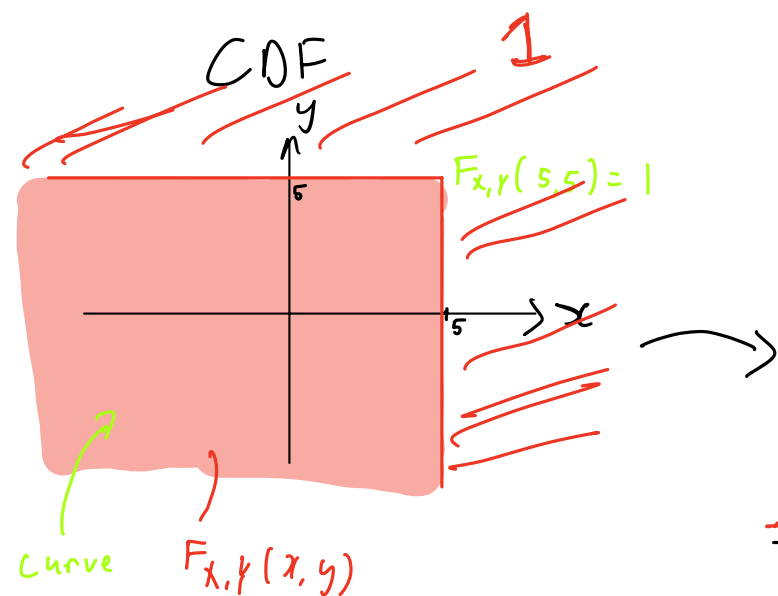
$P_{X,Y}(x,y)$

CDF

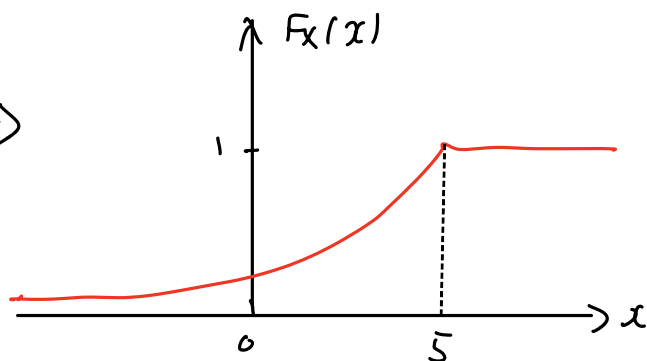


$F_{X,Y}(x,y)$

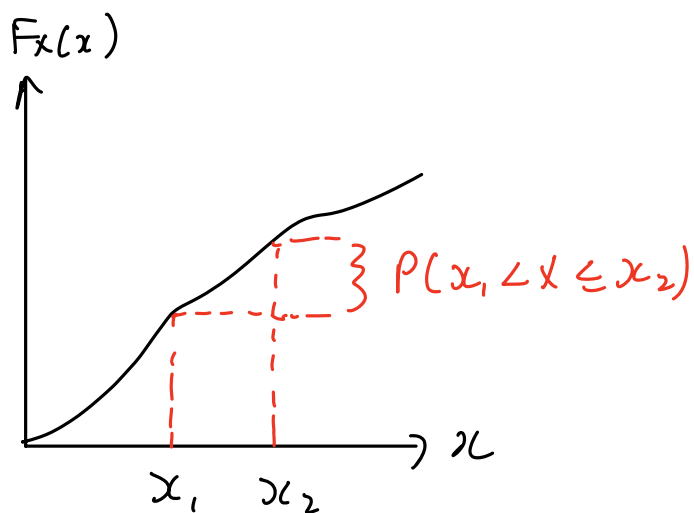
Marginalization



$$F_X(x) = F_{X,Y}(x, \infty)$$

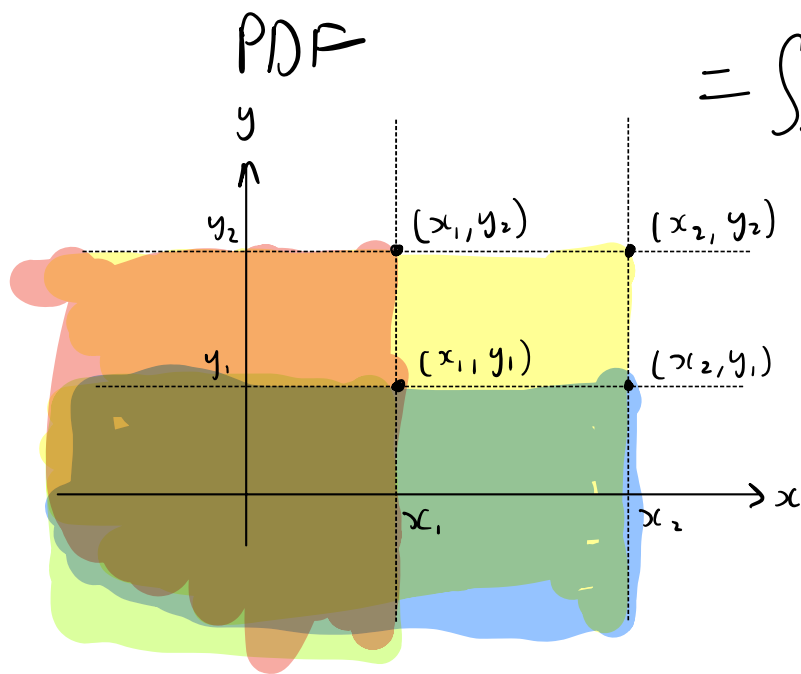


$$P(x_1 < X \leq x_2) = F_X(x_2) - F_X(x_1)$$



$$P(x_1 < X \leq x_2, y_1 < Y \leq y_2)$$

$$= F_{X,Y}(x_2, y_2) - F_{X,Y}(x_2, y_1) - F_{X,Y}(x_1, y_2) + F_{X,Y}(x_1, y_1)$$



$$= \int_{x_1}^{x_2} \int_{y_1}^{y_2} f_{X,Y}(x, y) dy dx$$

Example 5.2

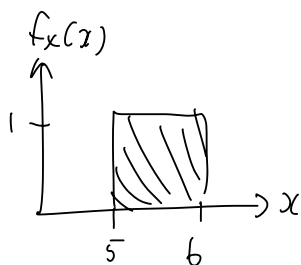
$$F_{X,Y}(x,y) = \begin{cases} 0 & x < 5, \\ 0 & y < 6, \\ (x-5)(y-6) & 5 \leq x < 6, 6 \leq y < 7, \\ y-6 & x \geq 6, 6 \leq y < 7, \\ x-5 & 5 \leq x < 6, y \geq 7, \\ 1 & \text{otherwise.} \end{cases}$$

$$F_X(x) = F_{X,Y}(x, \infty)$$

$$= \begin{cases} 0 & x < 5 \\ x-5 & 5 \leq x < 6 \\ 1 & \text{otherwise} \end{cases}$$

$$\downarrow \frac{d}{dx}$$

$$f_X(x) = \begin{cases} 1 & 5 \leq x < 6 \\ 0 & \text{otherwise} \end{cases}$$

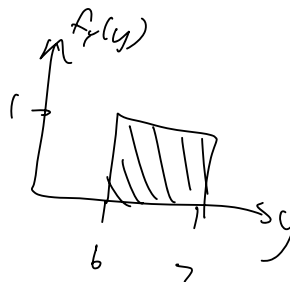


$$F_Y(y) = F_{X,Y}(\infty, y)$$

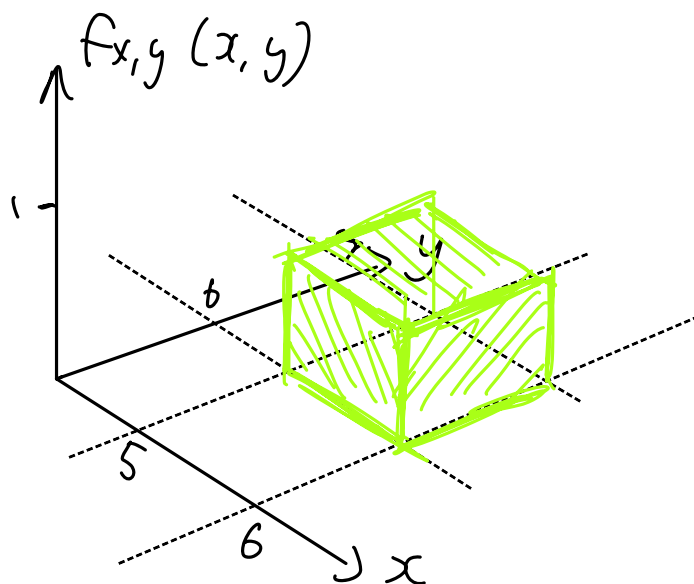
$$= \begin{cases} 0 & y < 6 \\ y-6 & 6 \leq y < 7 \\ 1 & \text{otherwise} \end{cases}$$

$$\downarrow \frac{d}{dy}$$

$$f_Y(y) = \begin{cases} 1 & 6 \leq y < 7 \\ 0 & \text{otherwise} \end{cases}$$



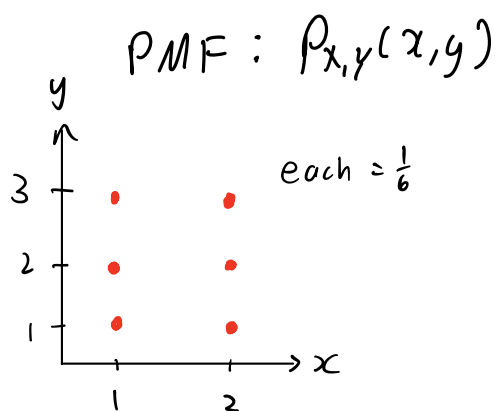
$$P_{X,Y}(x,y) = P(X=x, Y=y)$$



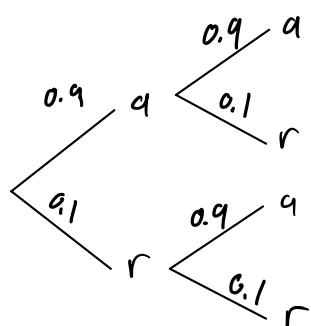
Discrete Marginal

$$P_x(x) = \sum_{y \in S_y} P_{x,y}(x,y)$$

$$= \begin{cases} \frac{1}{2} & x=1 \\ \frac{1}{2} & x=2 \\ 0 & \text{otherwise} \end{cases}$$



Example 5.3



$P_{x,y}(x,y)$			
outcomes	$P(\cdot)$	X	Y
aa	0,81	2	2
ar	0,09	1	1
ra	0,09	1	0
rr	0,01	0	0

$X = \#a$

$Y = \#a \text{ before first } r$

$P_{x,y}(x,y)$	$y=0$	$y=1$	$y=2$	$P_x(x)$
$x=0$	0,01	0	0	0,01
$x=1$	0,09	0,09	0	0,18
$x=2$	0	0	0,81	0,81
$P_y(y)$	0,1	0,09	0,81	1

Example 5.4

$$\begin{aligned} P(B) &= \sum_{(x,y) \in B} p_{X,Y}(x,y) \\ &= 0,01 + 0,09 + 0,81 \\ &= \underline{0,91} \end{aligned}$$

