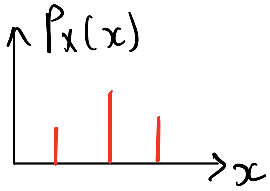
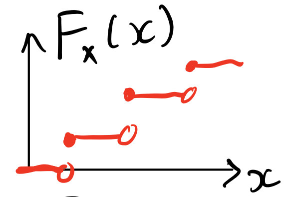
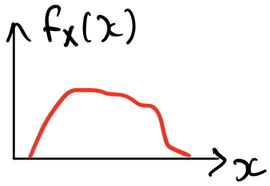
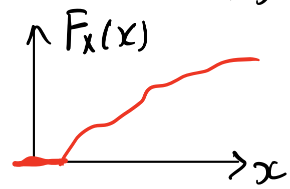
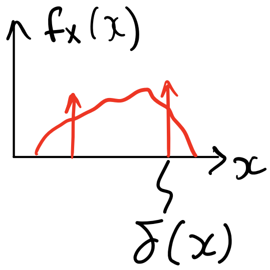
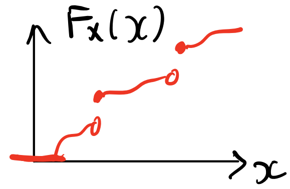
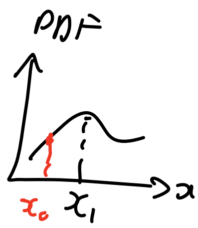
Mixed Random Variables

Discrete	PMF		CDF		Sums
Continuous	PDF		CDF		Integrals
Mixed	PDF		CDF		Integrals

$P(X = x_0) = 0.3$

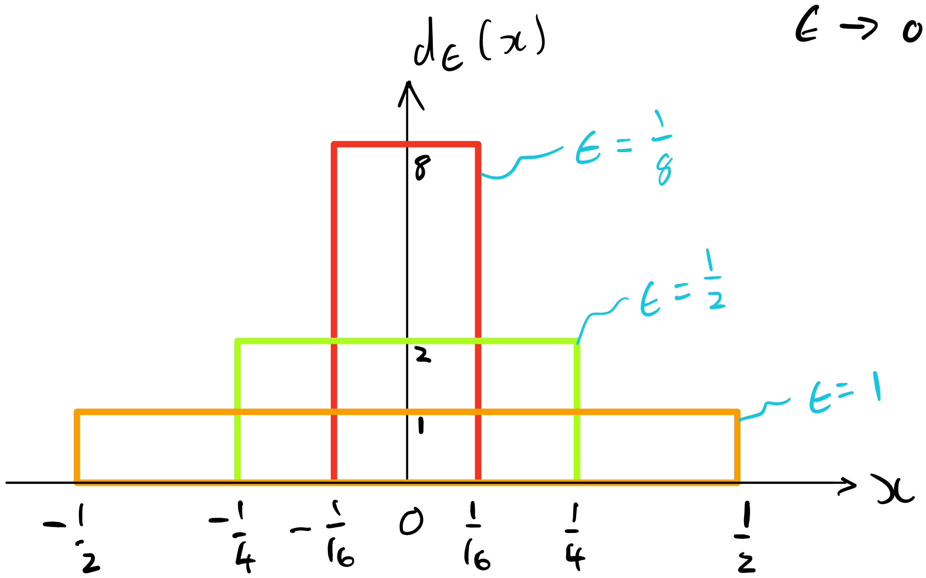
$P(X = x_1) = 0 \leftarrow$ not no prob.

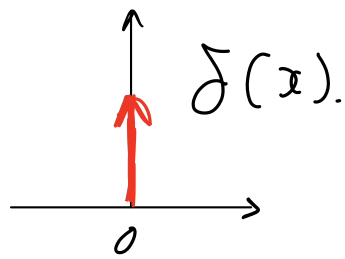


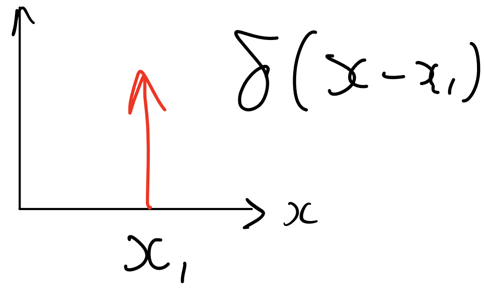
$$d_\epsilon(x) = \begin{cases} \frac{1}{\epsilon} & -\frac{\epsilon}{2} \le x \le \frac{\epsilon}{2} \\ 0 & \text{otherwise} \end{cases}$$

Uniform. Distribution?

$\epsilon \to 0 \quad d_\epsilon(x) \to \infty$







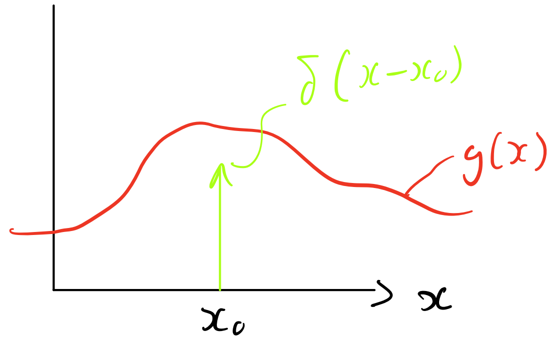
$$\int_{-\infty}^{\infty} \delta(x)\,dx = 1$$

Sifting Property

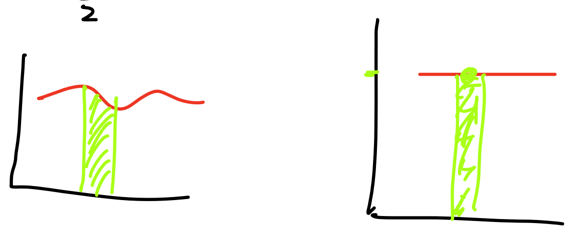
$$\int_{-\infty}^{\infty} g(x)\,\delta(x-x_0)\,dx = g(x_0)$$

$$g(x_0)\underbrace{\int_{x_0-\frac{\epsilon}{2}}^{x_0+\frac{\epsilon}{2}} d_\epsilon(x-x_0)\,dx}_{=1}$$

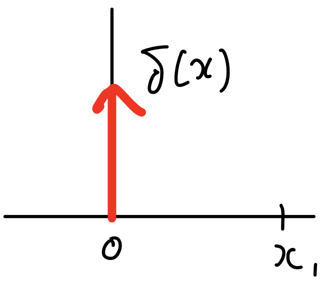
$$= g(x_0)$$



$$\int_{x_0-\frac{\epsilon}{2}}^{x_0+\frac{\epsilon}{2}} g(x)\, d_\epsilon(x-x_0)\,dx$$



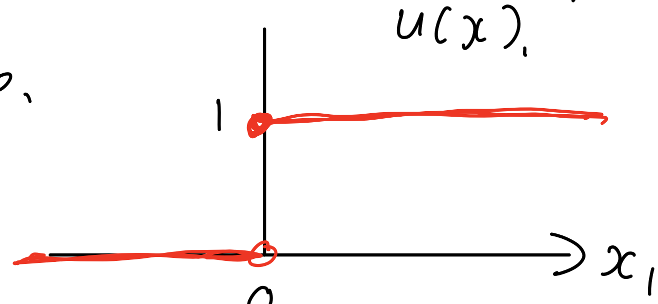
Integrating $\delta(x)$

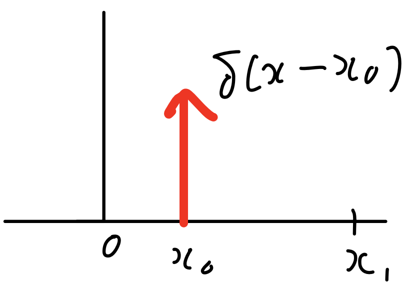


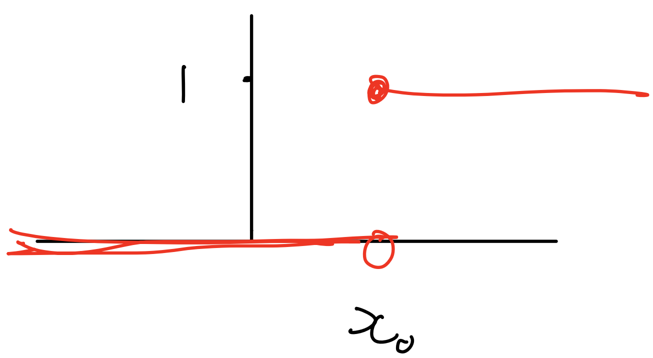
Integrate.

differentiate.

unit step.
$u(x)$.

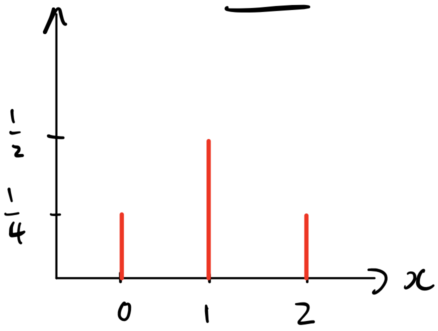




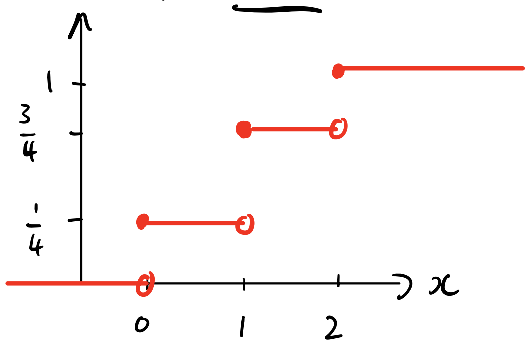


Blurring lines

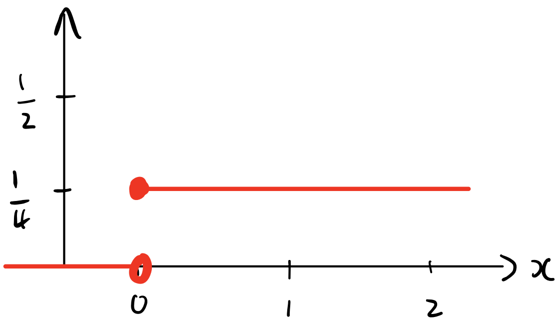
$p_X(x)$ PMF



$F_X(x)$ CDF



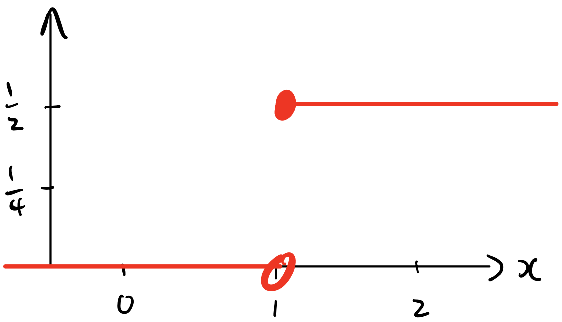
x_0:



$$F_X(x) = \sum_{x_i \in S_X} p_X(x_i) \cdot u(x - x_i)$$

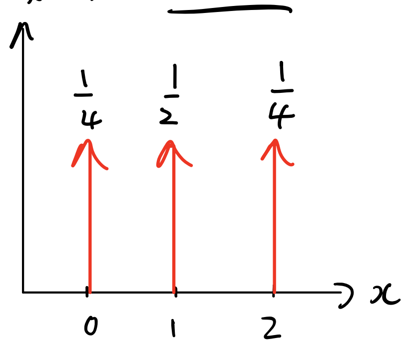
$$\frac{dF_X(x)}{dx}$$

x_1:

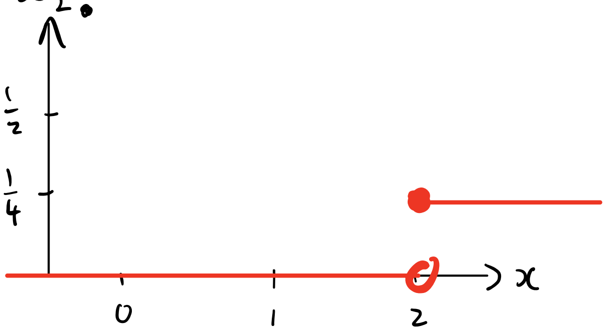


$$f_X(x) = \sum_{x_i \in S_X} p_X(x_i) \cdot \delta(x - x_i)$$

$f_X(x)$ PDF



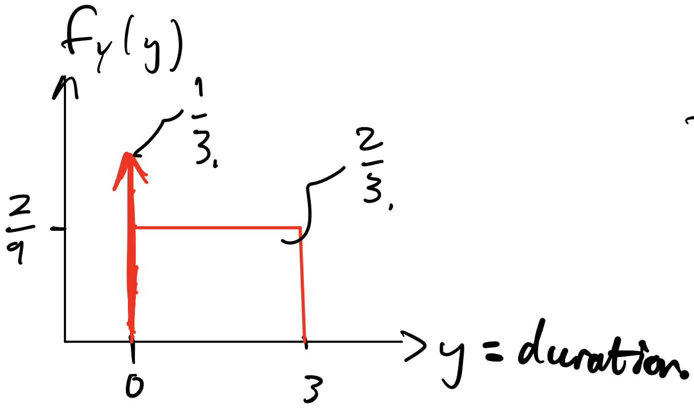
x_2:



$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx$$

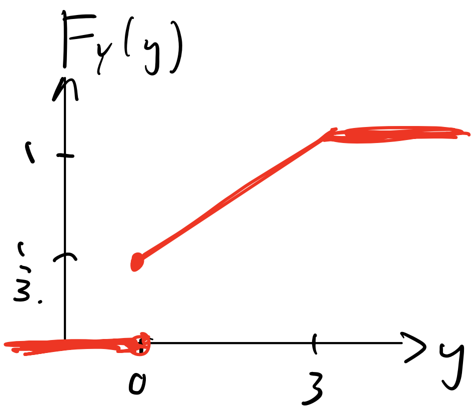
Example 4.17

$\frac{2}{3} = w \cdot h$
$= 3 \cdot h = ?$ $\quad h = \frac{2}{9}$



$$f_Y(y) = \begin{cases} \frac{1}{3}\delta(y) + \frac{2}{9} & 0 \le y < 3 \\ 0 & \text{otherwise} \end{cases}$$

Integrate $\int_{-\infty}^{y_1} f_Y(y)\,dy$.



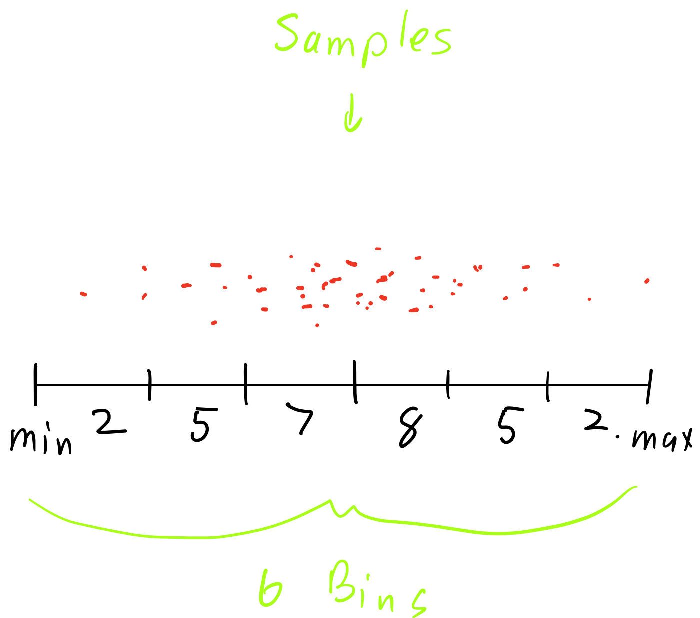
$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{3} + \frac{2}{9}y & 0 \le y < 3 \\ 1 & y \ge 3 \end{cases}$$

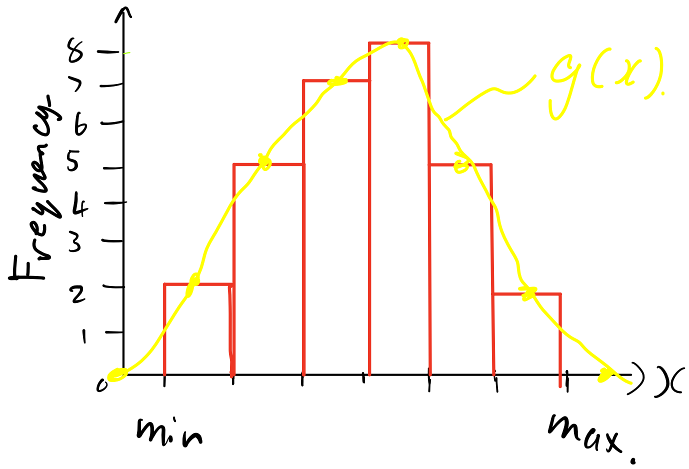
$$E[Y] = \int_{-\infty}^{\infty} y \cdot f_Y(y)\,dy$$

$$= \int_{-\infty}^{\infty} y \cdot \left(\frac{1}{3}\delta(y)\right) dy + \int_0^3 y \cdot \frac{2}{9}\,dy$$

$$= 0 + \frac{2}{9} \cdot \frac{1}{2} y^2 \Big|_0^3 = \frac{1}{9}(3)^2 = 1 \text{ min}$$

Histograms (estimating PDFs)





$$\frac{g(x)}{\int_{-\infty}^{\infty} g(x)\,dx} = f_X(x).$$