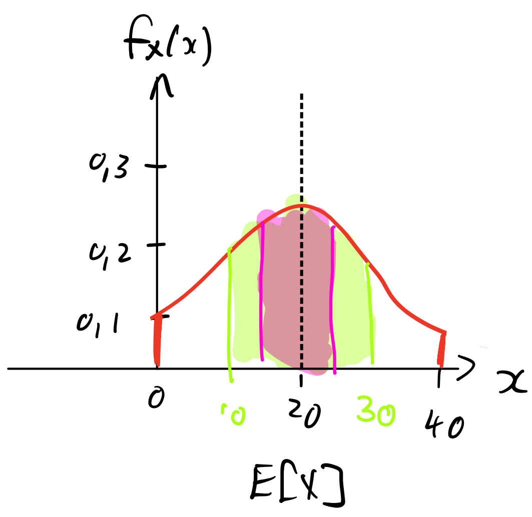
Continuous Random Variables.

Example:

X - temperature on 11 August 2025

$S_X = \{ x \mid 0 \le x \le 40 \}$

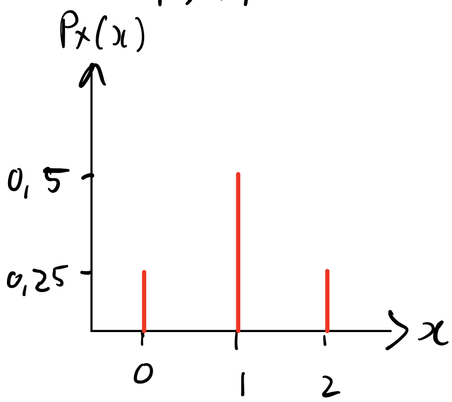


- $P(10 \le X \le 30) > P(15 \le X \le 25)$
- $P(X = 20) = 0$

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

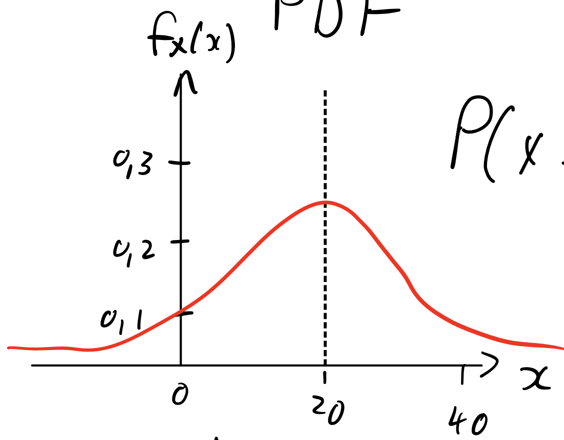
Discrete

PMF



Continuous

PDF

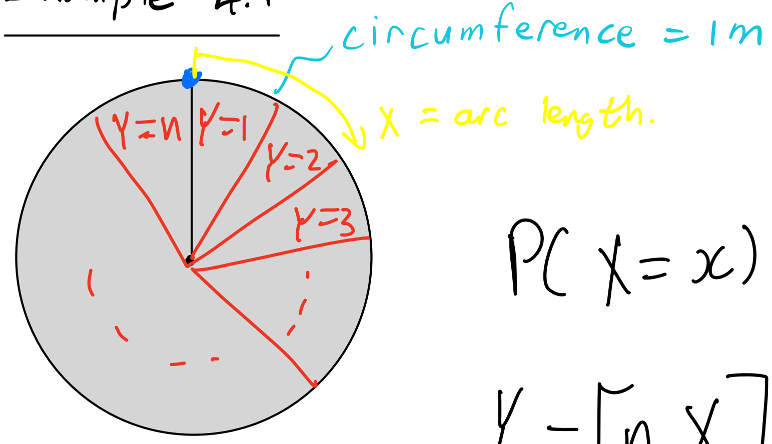


$P(X = x) = 0$

CDF

$F_X(x)$

Example 4.1



$$S_X = [0, 1)$$
$$= \{x \mid 0 \le x < 1\}$$

$$P(X = x) = ? \qquad \{X = x\} \subset \{Y = \lceil nx \rceil\}$$

$$Y = \lceil nX \rceil \qquad S_Y = \{1, 2, 3, \ldots n\}$$

$$P_Y(y) = \begin{cases} \frac{1}{n} & y = 1, 2, 3, \ldots n \\ 0 & \text{otherwise} \end{cases}$$

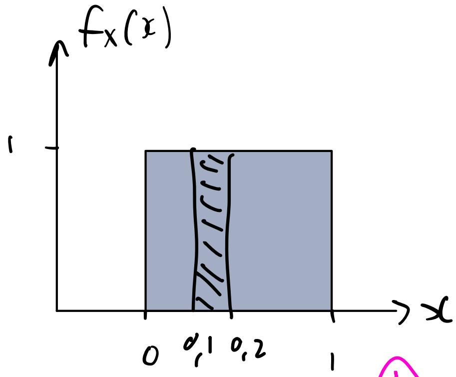
$$P(X = x) \le P(Y = \lceil nx \rceil) = \frac{1}{n}$$

$$P(X = x) \le \lim_{n \to \infty} P(Y = \lceil nx \rceil) = \left(\lim_{n \to \infty} \frac{1}{n} \right) = 0$$

$$\underbrace{P(X = x)}_{\ge 0} \le 0 \qquad A_1 \quad P(\cdot) \ge 0$$

$$P(X = x) = 0$$

Uniform Distribution

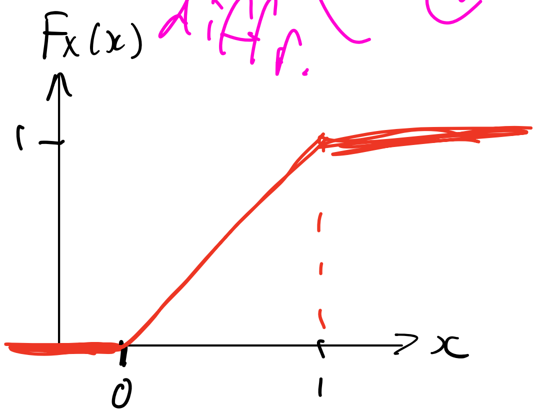


$$S_X = [0, 1)$$

$$f_X(x) = \begin{cases} 1 & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Integration

diffr.



$$F_X(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

Example 4.2

Quiz 4.2

a) $P(Y \leq -1) = 0$

b) $P(Y \leq 1) = 0{,}25 = F_Y(1)$

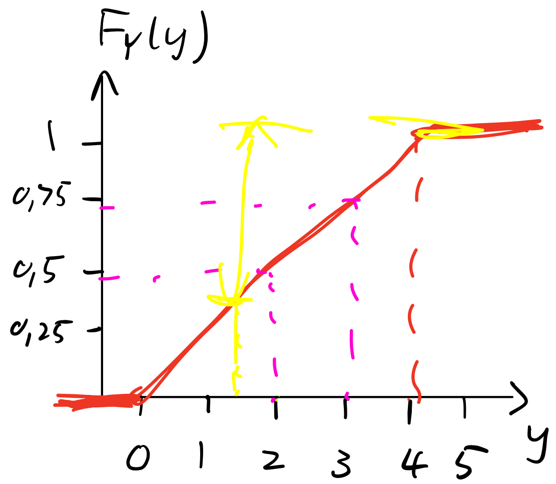
c) $P(2 < Y \leq 3) = F_Y(3) - F_Y(2)$

$= 0{,}75 - 0{,}5 = 0{,}25$

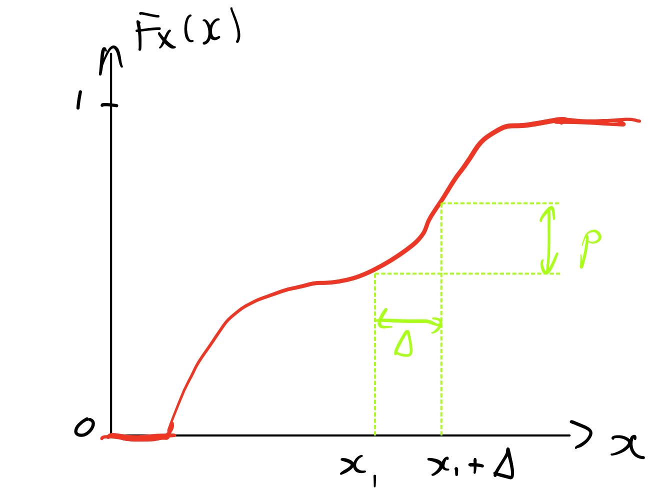
d) $P(Y > 1{,}5) = 1 - P(Y \leq 1{,}5)$

$= 1 - F_Y(1{,}5)$

$= 1 - \frac{1{,}5}{4} = \underline{0{,}625}$



PDF from CDF

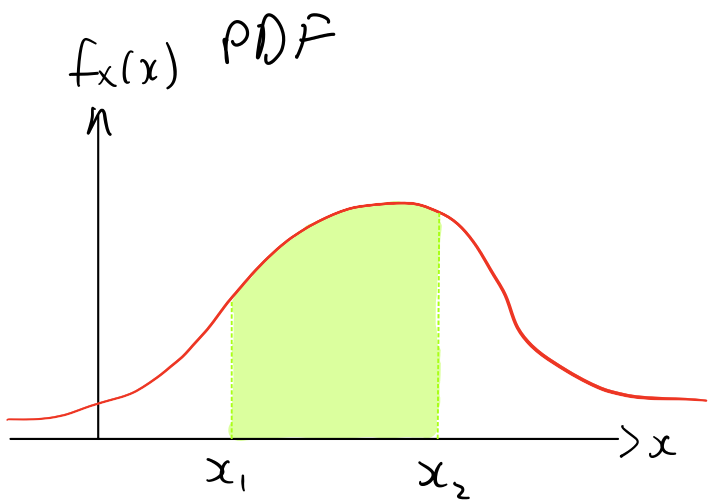


$$P(x_1 < X \le x_1 + \Delta) = \frac{F_X(x_1+\Delta) - F_X(x_1)}{\Delta} \cdot \Delta$$

$$\lim_{\Delta \to 0}$$

$$f_X(x) = \frac{dF_X(x)}{dx}$$

$$P(X = x_1) = \frac{dF_X(x_1)}{dx}$$
$$= 0$$



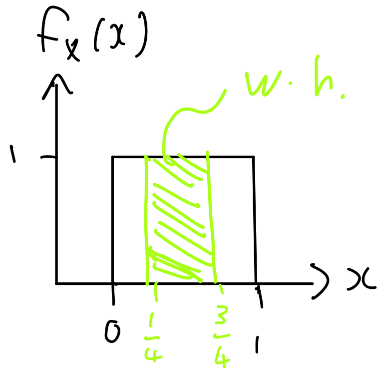
$$P(x_1 < X \le x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$$

amount of matter = density x Volume.

amount of Probability = density x (line/area/volume).
PDF

Example 4.4

$$f_X(x) = \begin{cases} 1 & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$



$w \cdot h. = \frac{1}{2} \cdot 1 = \frac{1}{2} =$

$$P\left(\frac{1}{4} < X \le \frac{3}{4}\right) = \int_{\frac{1}{4}}^{\frac{3}{4}} \overset{f_X(x)}{\downarrow}{1}\, dx$$

$$= x\Big|_{\frac{1}{4}}^{\frac{3}{4}} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}.$$

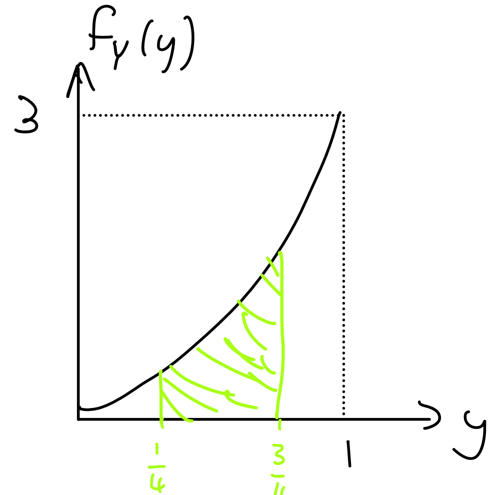
Example 4.5

$$f_Y(y) = \frac{d F_Y(y)}{dy} = \begin{cases} 3y^2 & 0 \le y < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P\left(\frac{1}{4} < X \le \frac{3}{4}\right)$$

$$= \int_{\frac{1}{4}}^{\frac{3}{4}} 3y^2\, dy$$

$$= y^3\Big|_{0{,}25}^{0{,}75} = \underline{0{,}40625} \longrightarrow$$



Expected Value

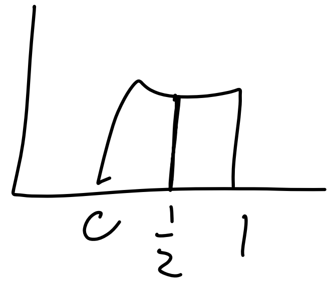
Spinner Example

$$f_X(x) = \begin{cases} 1 & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$$

$$= \int_0^1 x \cdot 1 \,dx$$

$$= \frac{1}{2} x^2 \Big|_0^1 = \frac{1}{2}$$



$$\sigma_X = \sqrt{Var[X]} = \sqrt{\frac{1}{12}}.$$

$$Var[X] = E[X^2] - E[X]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12}.$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_X(x)\,dx$$

$$= \int_0^1 x^2 \cdot 1 \,dx$$

$$= \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3}.$$

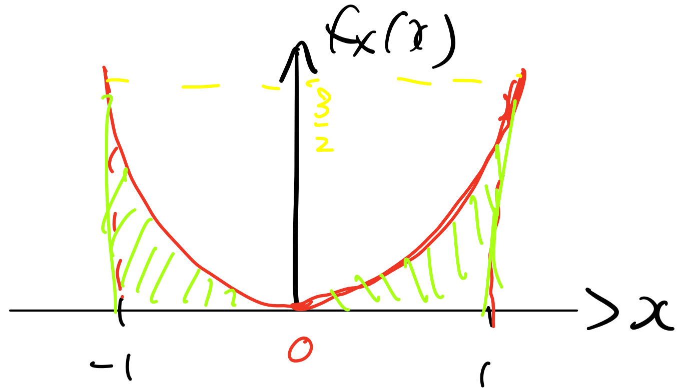
Example (based on Quiz 4.4)

$$f_X(x) = \begin{cases} \frac{3x^2}{2} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find c

b) Sketch PDF

c) Find σ_X $= 0.775$



a) $$\int_{-1}^{1} \frac{cx^2}{2}\, dx = 1$$

$$\frac{c}{2} \int_{-1}^{1} x^2\, dx = 1$$

$$\frac{c}{2} \left(\frac{1}{3} x^3 \Big|_{-1}^{1} \right) = 1$$

$$\frac{c}{2} \left(\frac{1}{3} + \frac{1}{3} \right) = 1$$

$$\frac{c}{3} = 1 \qquad c = 3.$$