Families of discrete random variables.

Example 3.10

Pascal distribution, but let's assume we didn't know.

$P(\text{reject}) = p$ $\quad$ $P(\text{accepted}) = 1-p.$

success

$L = X \sim$ num trials $\quad$ k - fixed of successes.

$k=4 \quad x=7$

$\underbrace{\boxed{raararr}}_{A}\underbrace{r}_{B}$ a r r a r

x^{th}

p

$6 = x-1$

$3 = k-1$ successes.

$$P_X(x) = P(X=x) = P(A \cap B)$$

$$= P(k-1 \text{ successes in } \overbrace{x-1}^{n} \text{ test} \wedge 1 \text{ success on } x^{th} \text{ test})$$

$$= P(A) \cdot P(B)$$

$$P(B) = p$$

$$P(A) = \text{Binomial}(n = x-1, p) \to Y$$

$$= P_Y(k-1) = \binom{x-1}{k-1} p^{k-1} (1-p)^{(x-1)-(k-1)}$$

$$P_X(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$$

Uniform distribution (k, ℓ)

eg. $k = 2$ $\quad \ell = 8$ (both inclusive)

$S_x = \{2, 3, 4, 5, 6, 7, 8\}$ $n = 7$

$n = \ell - k + 1$

$= 8 - 2 + 1 = 7$

Poisson

X - number of arrivals of thing of interest occurring λ times/s

When considering over time interval T (in seconds), then the rate is:

$$\alpha = \lambda T = \frac{\text{rate}}{s} \cdot s = \text{rate}$$ - unitless.

Example 3.13

$\lambda = 2$ hits/s

1. No hits in 0,25 s?
2. No more than 2 hits in 1s. $X \leq 2$

$$P_X(x) = \begin{cases} \dfrac{\alpha^x e^{-\alpha}}{x!} & x = 0, 1, 2 \ldots \\ 0 & \text{otherwise} \end{cases}$$

1. $\alpha = \lambda T = 2 \cdot 0{,}25 = 0{,}5$

$$P_X(x) = \frac{0{,}5^x e^{-0{,}5}}{x!}$$

$$P_X(0) = \frac{1 \, e^{-0{,}5}}{1} = e^{-0{,}5} = 60{,}7\%$$

2. $\alpha = \lambda T = 2 \cdot 1 = 2$

$$P_X(x) = \frac{2^x e^{-2}}{x!}$$

$$P(X \leq 2) = F_X(2) = P_X(0) + P_X(1) + P_X(2)$$
$$= 67{,}7\%$$

Example 3.14