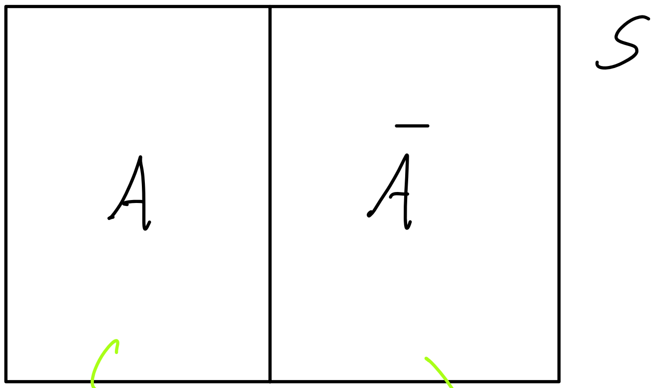
# Families of Discrete Random Variables.




$P(A) = p$ $\quad$ $P(\bar{A}) = 1 - p$

## Example 3.9

### Binomial

question? — RV $k = [0, n]$

$K$ is number of rejects in $n$ tests.

$\underbrace{n \text{ tests}}_{\text{Constant.}}$

$P(\text{rejected}) = p$

$P(\text{accepted}) = 1 - p$

$k=2$

What is the probability of seeing 2 rejected circuits if I test 3 circuits? $n=3$

$n = 3$ $k = 2$ Success = reject $(r)$ fail = accepted $(a)$

$P(r) = p$ $P(a) = 1-p$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| aaa | aar | ara | arr | raa | rar | rra | rrr | S |

$P(arr \cup rar \cup rra) = P(arr) + P(rar) + P(rra)$

$= (1-p)\,p \cdot p + p \cdot (1-p) \cdot p + p \cdot p \cdot (1-p)$

$= 3\,p^2(1-p)$

$\binom{n}{k} p^k (1-p)^{n-k}$

## Example

Binomial: eg. $n = 5$ $\quad P(\text{seeing } x \text{ } 1^s) = ?$

| $x$ | 2 | 1 | 3 | 2 | 2 | |
|---|---|---|---|---|---|---|
| | 1 0 0 1 0 | 1 0 0 0 0 | 1 1 0 1 0 | 1 0 0 0 1 | 0 1 0 1 0 | . . . . . |

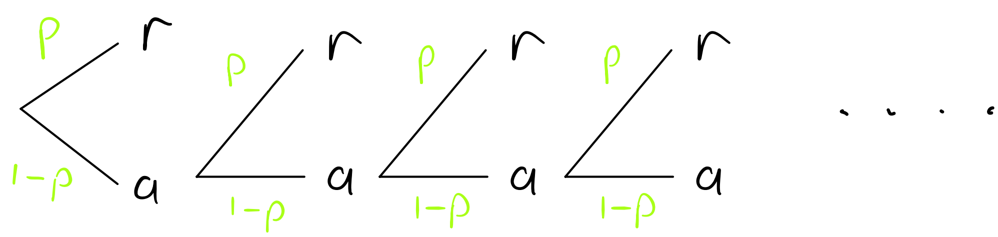
Geometric:

first success

| | 1 | 0 0 1 | 0 1 | 0 0 0 0 1 | 1 | 0 1 | 0 1 | 0 0 0 1 | 0 1 | 0 1 | 0 . . . . . |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 1 | 3 | 2 | 5 | 1 | 2 | 2 | 4 | 2 | 2 | |

## Example 3.8

Geometric, but let's assume we didn't know.




$P(Y=1) = p$

$P(Y=2) = (1-p)\cdot p$

$P(Y=3) = (1-p)(1-p)\cdot p$

$$P_Y(y) = \begin{cases} p(1-p)^{y-1} & y = 1, 2, 3 \ldots \\ 0 & \text{otherwise} \end{cases}$$

$p = \frac{1}{4}$

$$P_Y(y) = \begin{cases} \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{y-1} & y = 1, 2, 3 \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$F_Y(y) = \sum_{j=1}^{y} P_Y(j) = \sum_{j=1}^{y} \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{j-1}$$

$$= \left(\frac{1}{4}\right) \sum_{j=1}^{y} \left(\frac{3}{4}\right)^{j-1}$$

$$\sum_{j=1}^{n} x^{j-1} = \frac{(1-x^n)}{(1-x)}$$

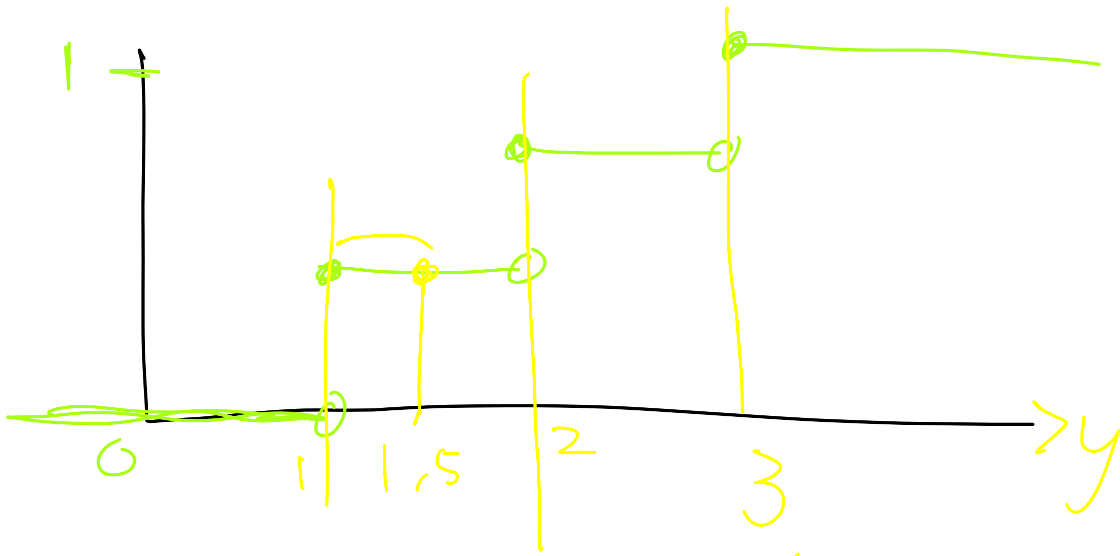
$$= \left(1 - \left(\frac{3}{4}\right)^{y}\right)$$

$$\underbrace{\underbrace{(1-x)}\sum_{j=1}^{n} x^{j-1}} = \underbrace{(1-x^n)}$$

$$= \left(1 - \left(\frac{3}{4}\right)^{\lfloor y \rfloor}\right)$$

$$\frac{1}{4} = 1-x$$

$$x = \frac{3}{4}$$




$$F_Y(y) = \begin{cases} 0 & y < 1 \\ 1 - \left(\frac{3}{4}\right)^{\lfloor y \rfloor} & y \geq 1 \end{cases}$$