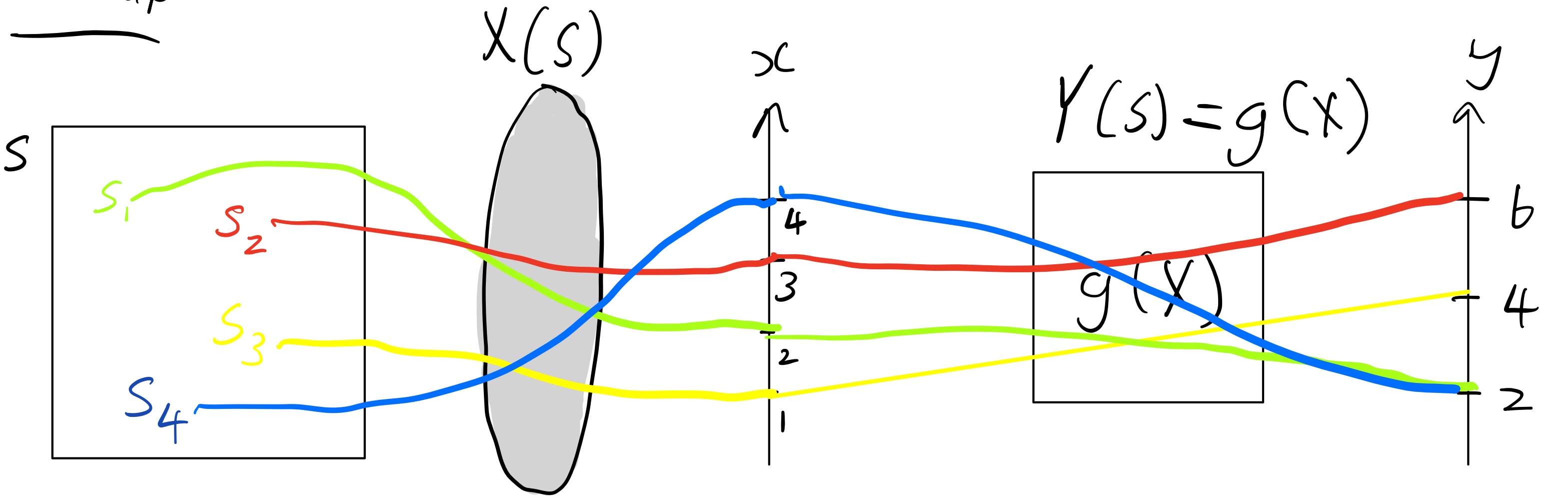
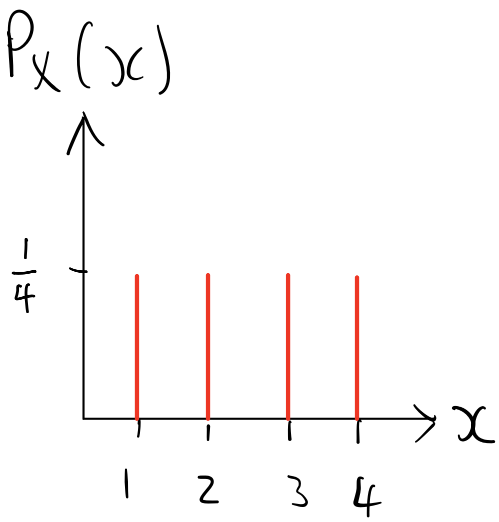
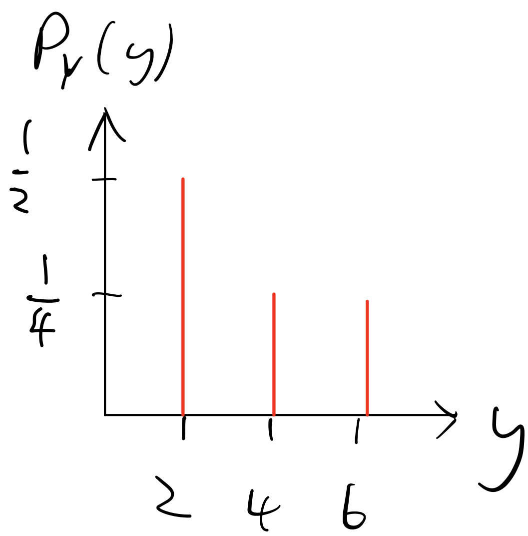
Lecture 7

Recap



$S_X = \{1, 2, 3, 4\}$ $\qquad$ $S_Y = \{2, 4, 6\}$





$$E[Y] = \sum_{y \in S_Y} y \cdot p_Y(y) = E[g(X)] = \sum_{x \in S_X} g(x) \cdot p_X(x)$$

$$= \frac{1}{4}(4+6) + \frac{1}{2}(2)$$

$$= 3,5$$

$$= \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 6 + \frac{1}{4} \cdot 2$$

$$= 3,5$$

Properties

$$\bullet\ E[\underbrace{aX+b}_{g(X)}] = \sum_{x \in S_X} \overbrace{(ax+b)}^{g(x)} \cdot P_X(x)$$

$$= \sum_{x \in S_X} \left[ax P_X(x) + b P_X(x)\right]$$

$$= \sum_{x \in S_X} \left(ax P_X(x)\right) + \sum_{x \in S_X} b P_X(x)$$

$$= a \underbrace{\sum_{x \in S_X} x \cdot P_X(x)} + b \underbrace{\sum_{x \in S_X} P_X(x)}_{1}$$

$$= a E[X] + b$$

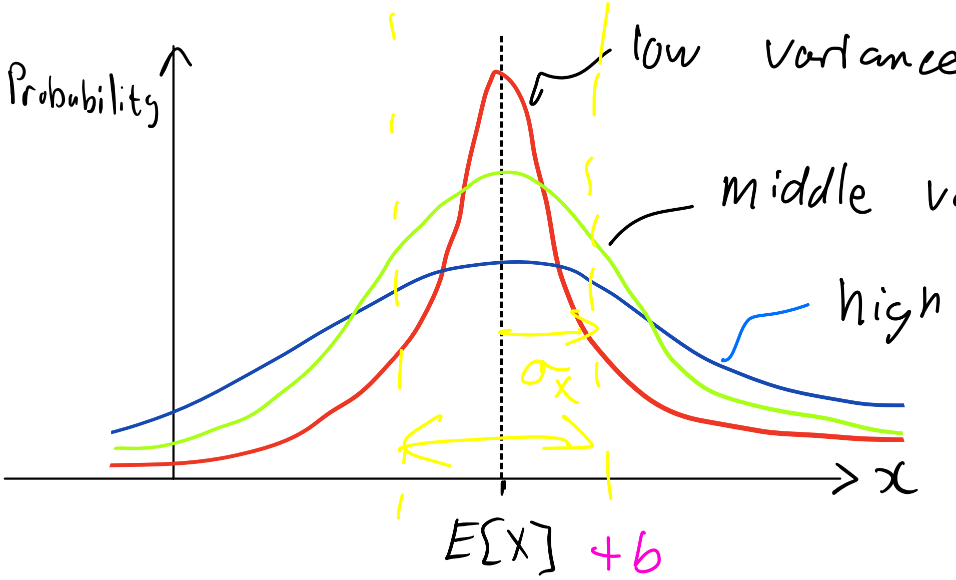
Constant $\quad a = 1 \quad b = -\mu_X$

$$E[X - \mu_X] = E[X] - \mu_X \qquad \mu_X = E[X]$$

$$= E[X] - E[X]$$

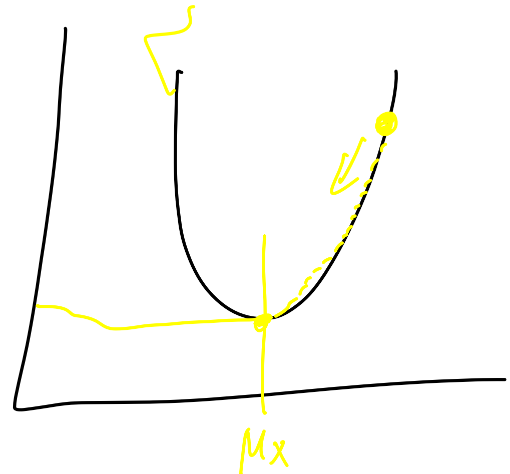
$$= 0$$

Variance



$$\sqrt{m^2} = \sigma_X$$

m.



$$Var[X] = E[\overbrace{(X-\mu_X)^2}^{g(x)}]$$

$$= \sum_{x \in S_X} \overbrace{(x-\mu_X)^2}^{g(x)} \cdot P_X(x)$$

$$= \sum_{x \in S_X} (x^2 - 2\mu_X x + \mu_X^2) \cdot P_X(x)$$

$$= \sum_{x \in S_X} \left[x^2 P_X(x) - 2\mu_X x \cdot P_X(x) + \mu_X^2 P_X(x)\right]$$

$$= \underbrace{\sum_{x \in S_X} \overbrace{x^2}^{g(x)} P_X(x)}_{E[X^2]} - 2\mu_X \underbrace{\sum_{x \in S_X} x \cdot P_X(x)}_{E[X]} + \mu_X^2 \cancel{\sum_{x \in S_X} P_X(x)}^{1}$$

$$= E[X^2] - 2\mu_X \overbrace{E[X]}^{\mu_X} + \mu_X^2$$

$$= E[X^2] - \mu_X^2 = E[X^2] - E[X]^2$$

Properties of Variance

Prove: $Var[aX+b] = a^2 Var[X]$

Rember: $Var[Y] = E[Y^2] - E[Y]^2$

Now set $Y = aX+b$

$$Var[aX+b] = E[(aX+b)^2] - E[aX+b]^2$$

$$= E[a^2X^2 + 2abX + b^2] - E[aX+b]^2$$

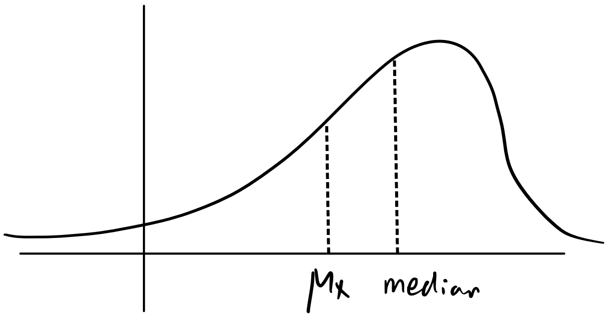
$$= a^2 E[X^2] + 2ab E[X] + b^2$$
$$- [aE[X] + b]^2$$

$$= a^2 E[X^2] + \cancel{2ab E[X]} + \cancel{b^2}$$
$$- [a^2 E[X]^2 + \cancel{2abE[X]} + \cancel{b^2}]$$

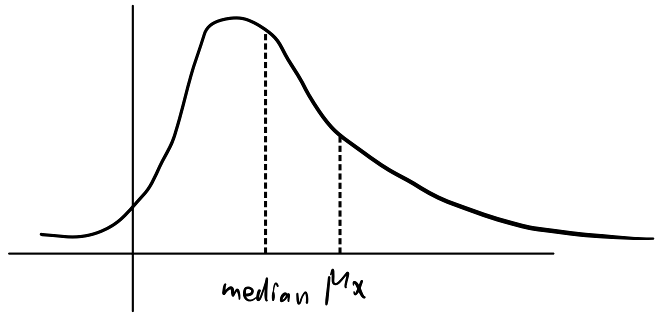
$$= a^2 [E[X^2] - E[X]^2]$$

$$= a^2 Var[X]$$

3. Skew $E[(X-\mu_X)^3]$

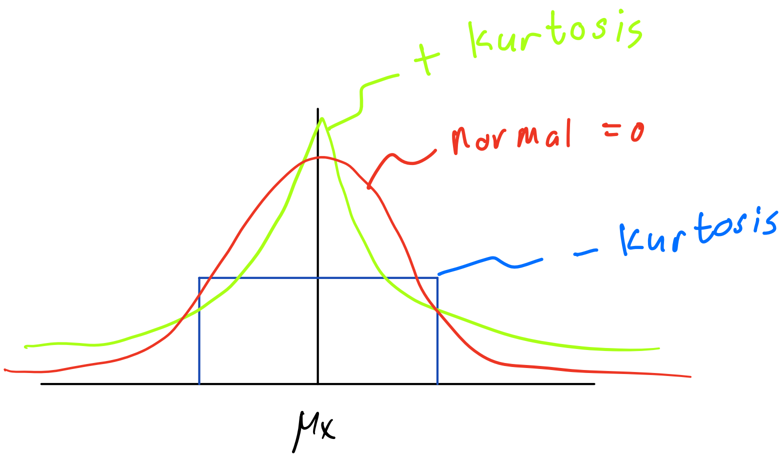


Negative skew
(left skewed)



Positive skew
(Right skew)

4. Kurtosis $E[(X-\mu_X)^4]$



Example 3.25 $\quad E[(\overset{g(x)}{X-\mu_X})^2]$

$$Var[X] = \sum_{x \in S_X} (x-\mu_X)^2 \, P_X(x) \qquad E[X] = \mu_X = 1$$

$$= (0-1)^2 \cdot \frac{1}{4} + (1-1)^2 \cdot \frac{1}{2} + (2-1)^2 \cdot \frac{1}{4}$$

$$= \frac{1}{2}$$

Example 3.27 $\quad \boxed{Var[aX] = a^2\, Var[X]}$

$$Var[V] = \sum_{v \in S_V} (v-\mu_v)^2 \cdot P_V(v) = \underbrace{E[V^2]} - \cancel{E[V]^2}$$

with $\mu_v = E[V] = \sum_{v \in S_V} v \cdot P_V(v)$

$$= \frac{1}{7}\left[(-3)+(-2)+(-1)+0+1+2+3\right]$$

$$= 0$$

$$E[V^2] = \sum_{v \in S_V} v^2 \cdot P_V(v)$$

$$= \frac{1}{7}\left((-3)^2+(-2)^2+(-1)^2+0+1^2+2^2+3^2\right)$$

$$= \frac{28}{7} = 4\ V^2 \quad \Rightarrow Var[V]$$

$$\sigma_v = \sqrt{Var[V]} = 2\ V$$

$$U = 1000 \cdot \underline{V} \quad (\text{mV.})$$

$$Var[U] = Var[1000 \cdot V]$$

$$= 1000^2 \, Var[V]$$

$$= 1000\,000 \cdot 4$$

$$= 4\,000\,000 \ \text{mV}^2.$$

$$\boxed{Var[aX] = a^2\, Var[X]}$$

$$\sigma_U = a\,\sigma_V = 1000 \cdot 2 = 2000 \ \text{mV}.$$