Example 3.5

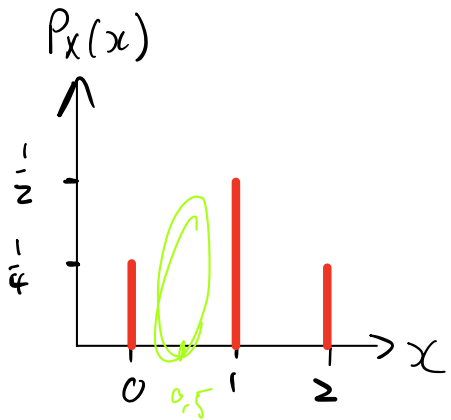
Outcomes	bb	bg	gb	gg
$P(\cdot)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
X = Points	0	1	1	2

$$S_X = \{0, 1, 2\}$$

$$P_X(x) = \begin{cases} \frac{1}{4} & x = 0 \\ \frac{1}{2} & x = 1 \\ \frac{1}{4} & x = 2 \\ 0 & \text{otherwise} \end{cases}$$

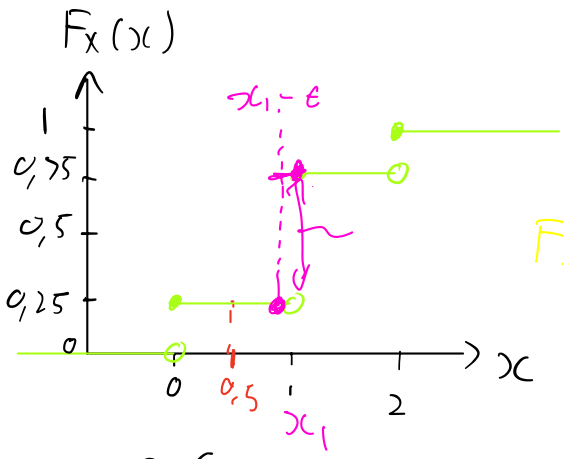
x	0	1	2
$P_X(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

PMF



$P(X = x)$

CDF



$F_X(1,5) = 0,25$

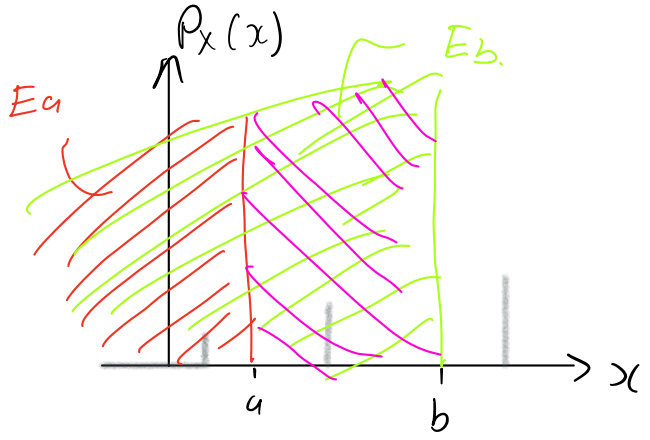
$P(X \leq x)$

$F_X(0) = 0,25$

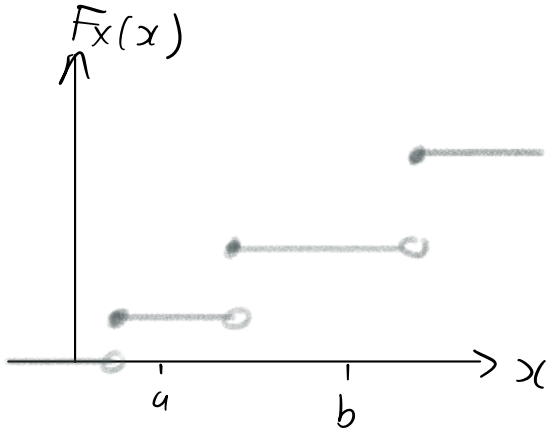
$P(X = 0,5) = P_X(0,5) = 0$

$P(X \leq 0,5) = F_X(0,5) = 0,25$

$b \geq a$: $F_X(b) - F_X(a) = P(a < X \leq b)$



PMF



CDF

$E_a = \{X \leq a\}$

$E_b = \{X \leq b\}$

$E_{ab} = \{a < X \leq b\}$

$E_b = E_a \cup E_{ab}$

$P(E_b) = P(E_a \cup E_{ab})$

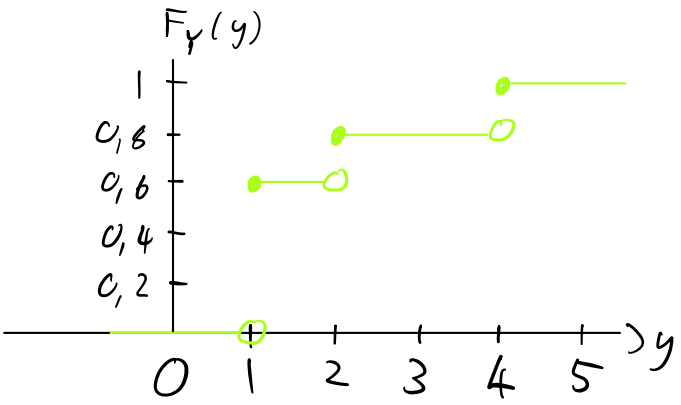
$= P(E_a) + P(E_{ab})$

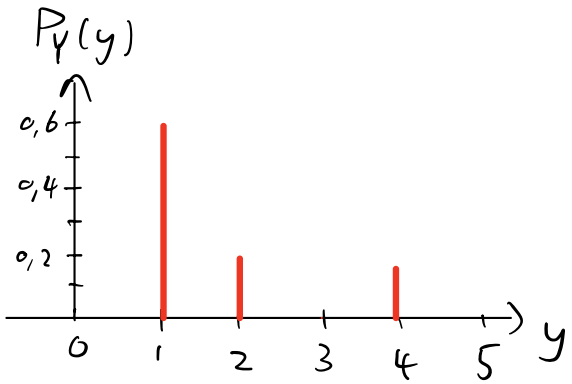
$P(E_{ab}) = P(E_b) - P(E_a)$

$P(a < X \leq b) = P(X \leq b) - P(X \leq a)$

$= F_X(b) - F_X(a)$

Quiz 3.4





a) $P(Y<1) = 0 = P(Y \leq 0)$

b) $P(Y \leq 1) = F_X(1) = 0,6$

c) $P(Y>2) = 1 - P(Y \leq 2) = 1 - F_X(2) = 0,2$

d) $P(Y \geq 2) = 1 - P(Y<2) = 1 - 0,6 = 0,4$

e) $P(Y=1) = 0,6$

f) $P(Y=3) = 0 = P_Y(3)$

$P(Y \leq 3) = X = F_X(3)$

Averages

1 2 2 5 7 8 $\quad S_X = \{1, 2, 5, 7, 8\}$

mean: $1+2+2+5+7+8 = 25$ $\quad \frac{25}{6} = 4\frac{1}{6}$

mode: 2

median: $\frac{2+5}{2} = 3{,}5$

$$m_n = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}\sum_{x \in S_X} x \cdot N_x = \sum_{x \in S_X} x \cdot \frac{N_x}{n}$$

$\frac{N_x}{n}$: $P(X=x)$, $P_X(x)$

$$= \sum_{x \in S_X} x \cdot P_X(x)$$

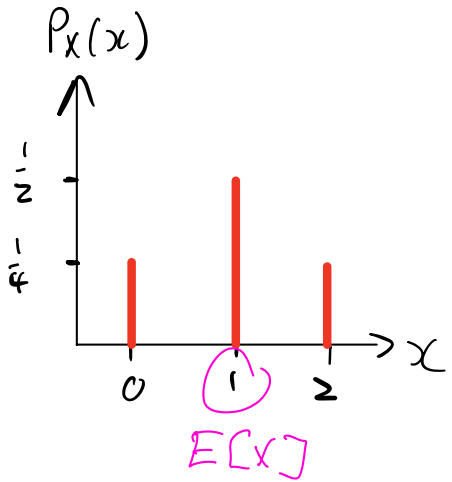
$X = 10 - 2(Y)$, $g(Y)$

Basketball example

$$E[X] = \sum_{x \in S_X} x \cdot P_X(x)$$

$$= 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4}$$

$$= 1$$



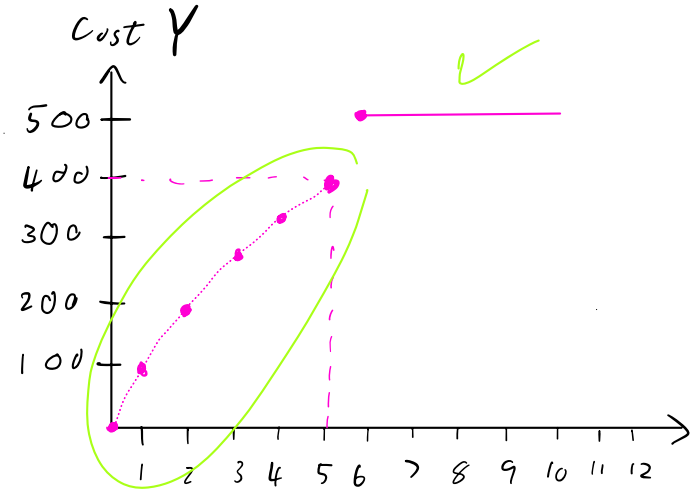
Example 3.19

- \$1 first pound
- \$0.9 second pound
- ⋮
- \$0.6 fifth pound
- 6–10 pounds = \$5
- > 10 Pounds not accepted

charge in cents

$Y = g(X)$

$Y = g(X)$



$$g(x) = \begin{cases} 105x - 5x^2 & x = 1, 2, 3, 4, 5 \\ 500 & x = 6, 7, 8, 9, 10 \end{cases}$$

$X \in [0, 10]$

$ax^2 + bx + c \qquad c = 0$

$ax^2 + bx = y$

$a(1)^2 + b(1) = 100$

$a(5)^2 + b(5) = 400$

Example 3.20

$$P_X(x) = \begin{cases} \frac{1}{4} & x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

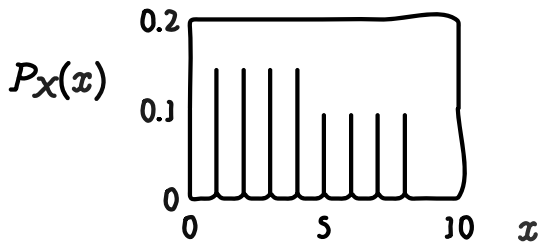
$$P_Y(y) = \begin{cases} \frac{1}{4} & y = 100, 190, 270, 340 \\ 0 & \text{otherwise.} \end{cases}$$

$$E[Y] = \sum_{y \in S_Y} y \cdot P_Y(y)$$

$$= (100 + 190 + 270 + 340) \cdot \frac{1}{4}$$

$$= 225$$

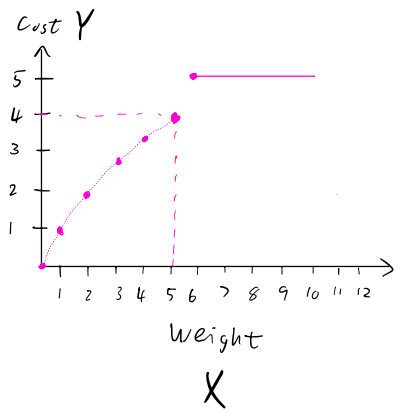
Example 3.21

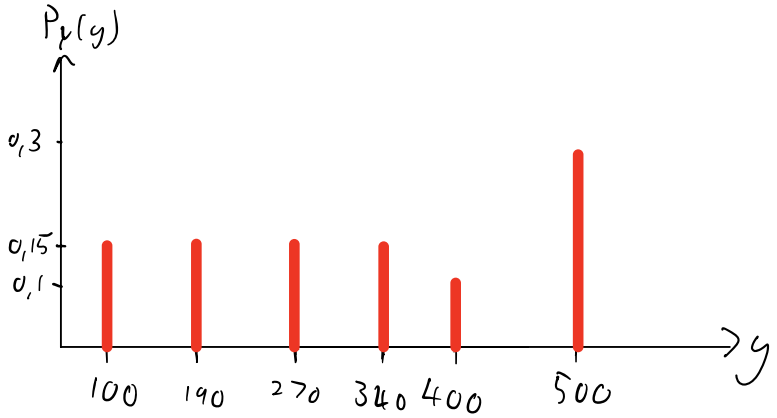


$$P_X(x) = \begin{cases} 0.15 & x = 1, 2, 3, 4, \\ 0.1 & x = 5, 6, 7, 8, \\ 0 & \text{otherwise.} \end{cases}$$

$$Y = 105X - 5X^2$$

$$P_Y(y) = \begin{cases} 0,15 & y = 100, 190, 270, 340 \quad (x = 1, 2, 3, 4) \\ 0,1 & y = 400 \quad (x = 4) \\ 0,3 & y = 500 \quad (x = 6, 7, 8) \\ 0 & \text{otherwise} \end{cases}$$





$$E[Y] = 0,15(100 + 190 + 270 + 340) + 0,1(400) + 0,3(500)$$

$$= 325 \text{ cents}$$