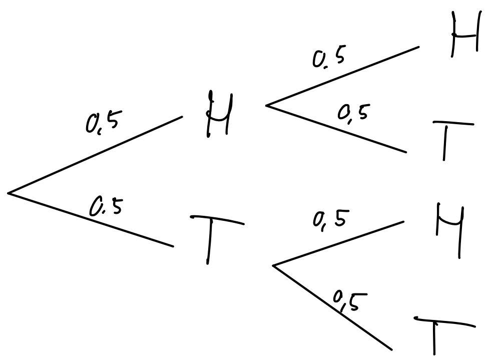
# Lecture 4

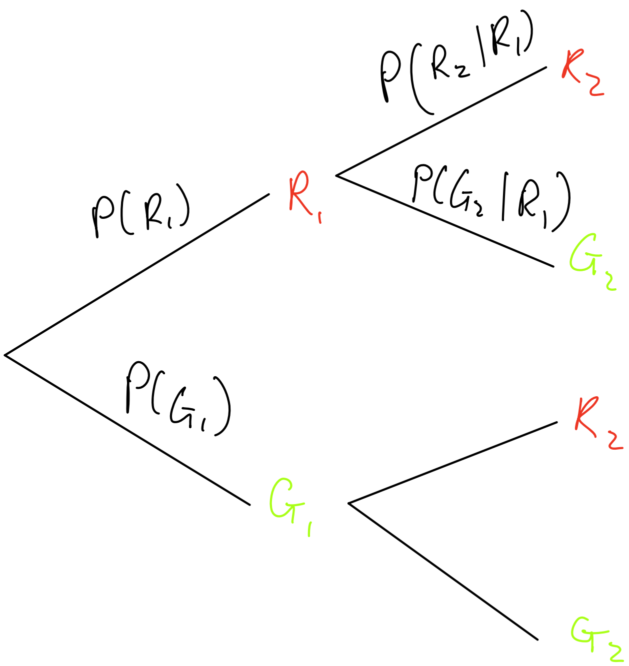
# Tree Diagrams




| Outcomes | Probability |
| --- | --- |
| HH | $P(H \cap H) = P(H) \cdot P(H)$ $= 0.5 \cdot 0.5 = 0{,}25$ |
| HT | $P(H \cap T) = P(H) \cdot P(T)$ $= 0{,}25$ |
| TH | $P(T \cap H) = 0{,}25$ |
| TT | $P(T \cap T) = 0{,}25$ |

## Without Replacement

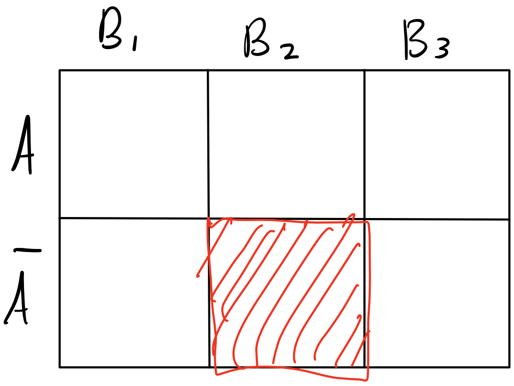
Red and green balls



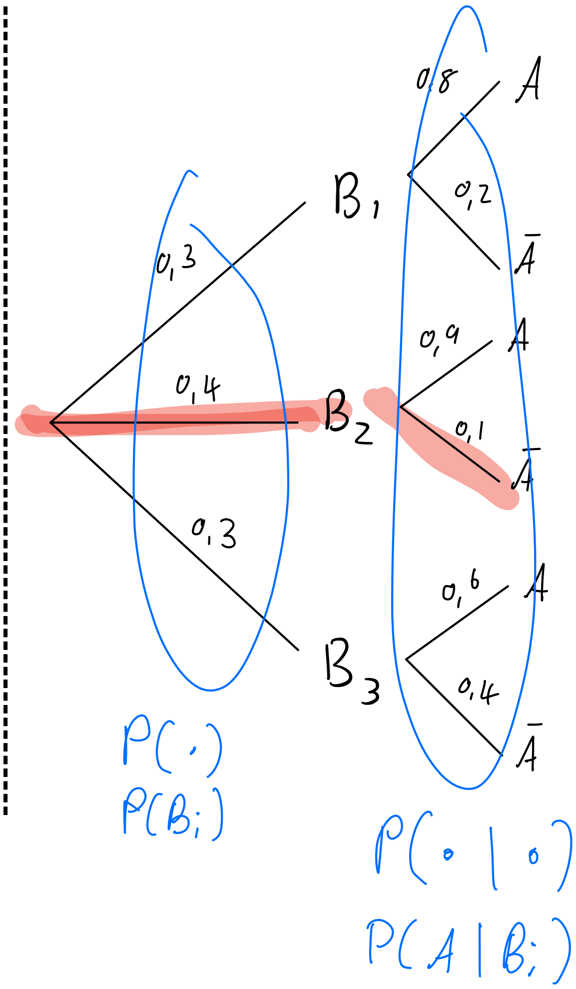

| Outcomes. | Probability |
| --- | --- |
| $R_1 R_2$ | $P(R_1 \cap R_2) = P(R_1) \cdot P(R_2 \mid R_1)$ |
| $R_1 G_2$ | $P(R_1 \cap G_2) = P(R_1) \cdot P(G_2 \mid R_1)$ |
| $G_1 R_2$ | $P(G_1 \cap R_2)$ |
| $G_1 G_2$ | $P(G_1 \cap G_2)$ |

# Example 3 Machines

| | | | |
|---|---|---|---|
| $B_1$ — 3000 | $P(B_1) = \frac{3}{10}$ | $P(A \mid B_1) = 0{,}8$ | $P(\bar{A} \mid B_1) = 0{,}2$ |
| $B_2$ — 4000 | $P(B_2) = \frac{4}{10}$ | $P(A \mid B_2) = 0{,}9$ | $P(\bar{A} \mid B_2) = 0{,}1$ |
| $B_3$ — 3000 | $P(B_3) = \frac{3}{10}$ | $P(A \mid B_3) = 0{,}6$ | $P(\bar{A} \mid B_3) = 0{,}4$ |
| 10 000 | | | |




2 Partitions




Outcomes

$B_1 \cap A$

$B_1 \cap \bar{A}$

$B_2 \cap A$

$P(B_2 \cap \bar{A}) = P(B_2) \cdot P(\bar{A} \mid B_2)$
$= 0{,}4 \cdot 0{,}1$
$= 0{,}04$

$B_3 \cap A$

$B_3 \cap \bar{A}$

$\Sigma = 1$

# Example Box with balls

- 6 red balls
- 9 green balls

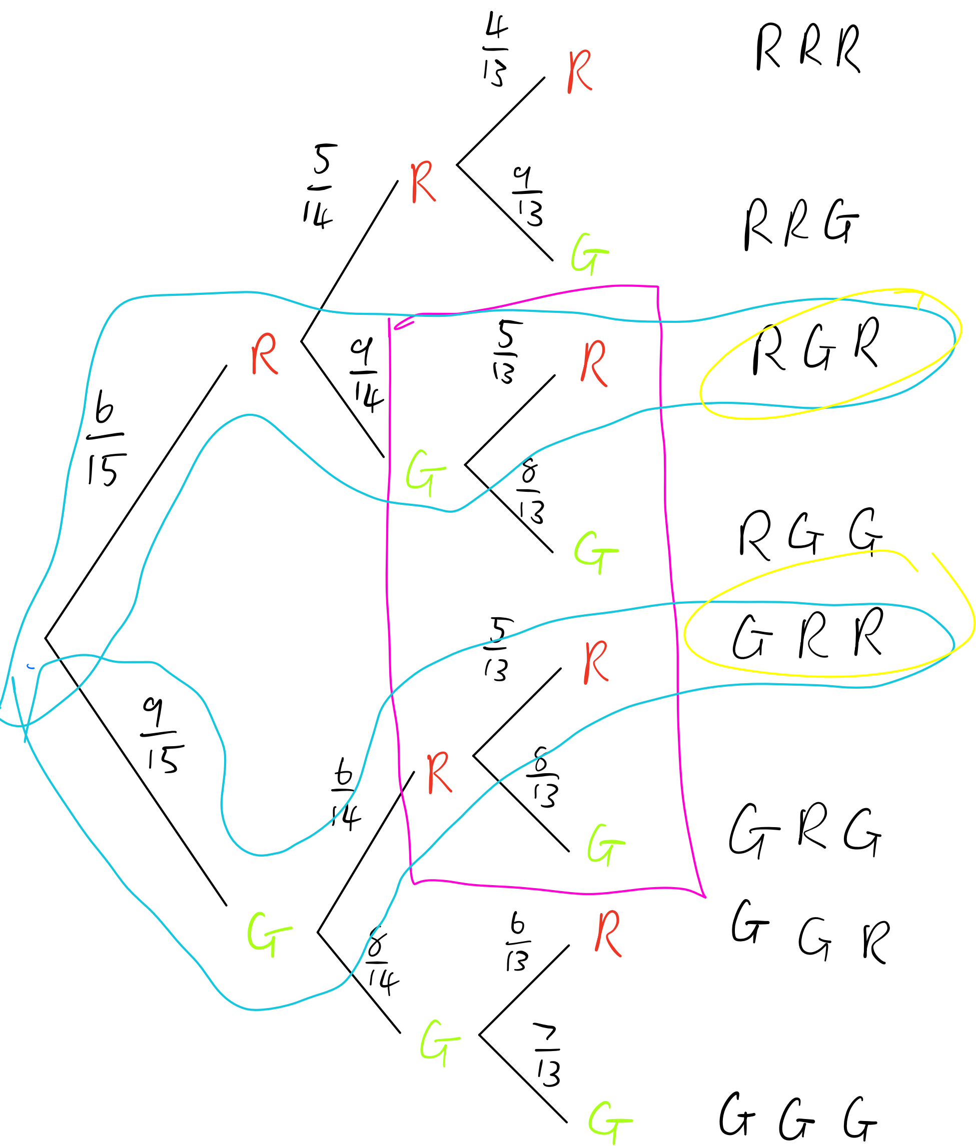
$\Big\}$ 15 in total

Draw 3 without replacement

- $P(3^{rd}\ R \mid \text{first 2 red}) = \frac{4}{13}$
- $P(3^{rd}\ R \mid \text{first 2 different}) = \frac{5}{13} = 0{,}385$
- $P(RGR) = \frac{6}{15} \cdot \frac{9}{14} \cdot \frac{5}{13} = \frac{9}{91} = 0{,}0989$
- $P(RGR \cup GRR) = P(RGR) + P(GRR) = \frac{18}{91} = 0{,}198$

ME

Outcomes:




$$\frac{10}{26} = \frac{5}{13}$$

| | Without Replacement | With Replacement |
|---|---|---|
| Order not important (sets) | Combinations $\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$ | Multinomial coefficient $\binom{r+N-1}{r} = \frac{(r+N-1)!}{(N-1)!\,r!}$ |
| Order is important (sequences) | Permutations $P^n_k = \frac{n!}{(n-k)!}$ | $n^k$ |

## Permutations Example

$n = 52 \quad k = 4$

$52 \times 51 \times 50 \times 49 = 6\ 497\ 400$

$n \times (n-1) \times (n-2) \times (n-3) \times \frac{\overset{48!}{(n-4)!}}{(n-4)!} = \frac{n \times (n-1) \cdots (2)(1)}{(n-4)(n-3) \cdots (2)(1)}$

$P^n_k = \frac{n!}{(n-k)!}$

# Combinations Example

A, B, C, D $\quad n=4 \quad k=2.$

AB, AC, AD, BC, BD, CD $= 6.$ Combinations

$n=2 \ k2.$

AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC $\Rightarrow 12.$

$$P^4_2 = \binom{4}{2} \cdot P^2_2 \rightarrow \binom{4}{2} = \frac{P^4_2}{P^2_2} = \frac{4!}{(4-2)! \cdot 2!}$$

Permutations

$$= \frac{n!}{(n-k)!\,k!}$$

$n = 3$

$\binom{3}{0} = 1$

$\binom{3}{1} = 3$

$\binom{3}{2} = 3$

$\binom{3}{3} = 1$

$n=5$

A, B, C, D, E $\quad k=2 \quad n-k=3$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{5}{2} = \binom{5}{3} \qquad \frac{n!}{(n-k)!\ k!} \Rightarrow \frac{5!}{3!\cdot 2!} = \frac{5!}{2!\cdot 3!}$$

AB, AC, AD, AE, BC, BD, BE, CD, CE, DE $= 10$

ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE $= 10$

# Example 2.9

Draw 7 cards. Probability of no queens?

$$P(\text{no queens}) = \left(\frac{48}{52}\right) \cdot \left(\frac{47}{51}\right) \cdots \left(\frac{42}{46}\right) = \ldots$$

$$= \frac{H_{NQ}}{H} = \frac{\binom{48}{7}}{\binom{52}{7}} = 0{,}5504$$

$H = \binom{52}{7}$ each $\frac{1}{H}$ likely.

$H_{NQ} = \binom{52-4}{7}$ set with no queens.

Draw 7 cards with replacement. Probability of no queens?

$52^7$ $\quad$ $48^7$ $\quad$ $P(\cdot) = \left(\frac{48}{52}\right)^7 = 0{,}5710$

$= P(\text{no queens on first try})^7$

# Example 2.17