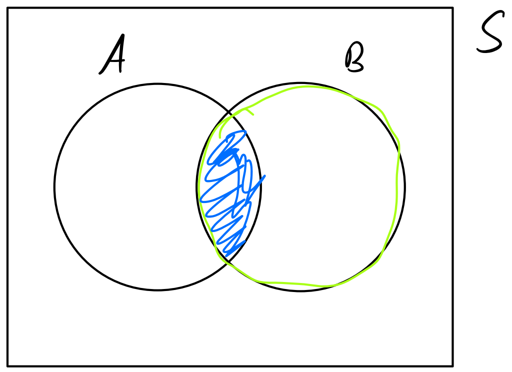
Recap



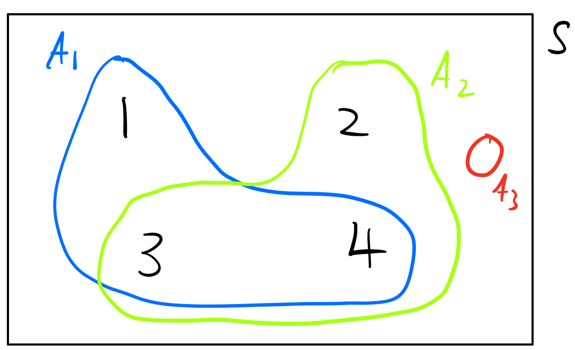
$P(A)$

$$P(A|B) = \frac{P(A \cap B)}{\underbrace{P(B)}_{\text{normalization}}}$$

entire sample space

$$P(A \cap B) = P(A|B) \cdot P(B)$$
$$= P(A) \cdot P(B)$$

Example 1.20



LHS   ?   RHS.

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3)$$
$$0 \quad = \quad \frac{3}{4} \cdot \frac{3}{4} \cdot 0$$

$$P(A_1 \cap A_2) \stackrel{?}{=} P(A_1)\, P(A_2)$$
$$\frac{1}{2} \neq \frac{3}{4} \cdot \frac{3}{4}$$
$$\frac{9}{16}$$

No, not mutually independent.

Total Probability

$$A = C_1 \cup C_2 \cup C_3 \cup C_4$$

$$\begin{aligned}
P(A) &= P(C_1 \cup C_2 \cup C_3 \cup C_4) \quad A3. \\
&= P(C_1) + P(C_2) + P(C_3) + P(C_4) \\
&= P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) + P(A \cap B_4) \\
&= \sum_{i=1}^{4} P(A \cap B_i)
\end{aligned}$$

Example 1.15

	Text	Image	Video
Long	0,3	0,12	0,15
Brief	0,2	0,08	0,15

- $P(\text{long and images}) = P(LI) = 0{,}12$.

$$P(L) = P(L \cap T) + P(L \cap I) + P(L \cap V)$$
$$= 0{,}3 + 0{,}12 + 0{,}15 = \underrightarrow{0{,}57}$$

Example 1.16

B_1	–	3000
B_2	–	4000
B_3	–	3000
		10 000

$P(B_1) = \frac{3}{10}$

$P(B_2) = \frac{4}{10}$

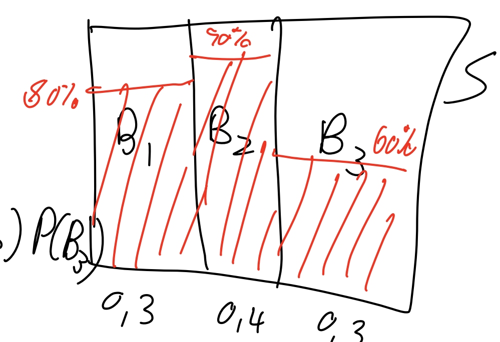
$P(B_3) = \frac{3}{10}$

$P(B_1 \text{ accept}) = 80\% = P(A \mid B_1)$

$P(B_2 \text{ accept}) = 90\% = P(A \mid B_2)$

$P(B_3 \text{ accept}) = 60\% = P(A \mid B_3)$

$$P(\text{acceptable}) = P(A) = ?$$



$$= P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$$
$$= P(A \mid B_1) \cdot P(B_1) + P(A \mid B_2)\, P(B_2) + P(A \mid B_3)\, P(B_3)$$
$$= 0{,}8 \cdot 0{,}3 + 0{,}9 \cdot 0{,}4 + 0{,}6 \cdot 0{,}3$$
$$= 0{,}78 = 78\%$$

Bayes' Theorem

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \qquad P(A\cap B) = P(B\cap A)$$

$$P(A|B)\,P(B) = P(A\cap B)$$

$$P(B|A)\,P(A) = P(B\cap A)$$

$$P(A|B)\,P(B) = P(B|A)\,P(A)$$

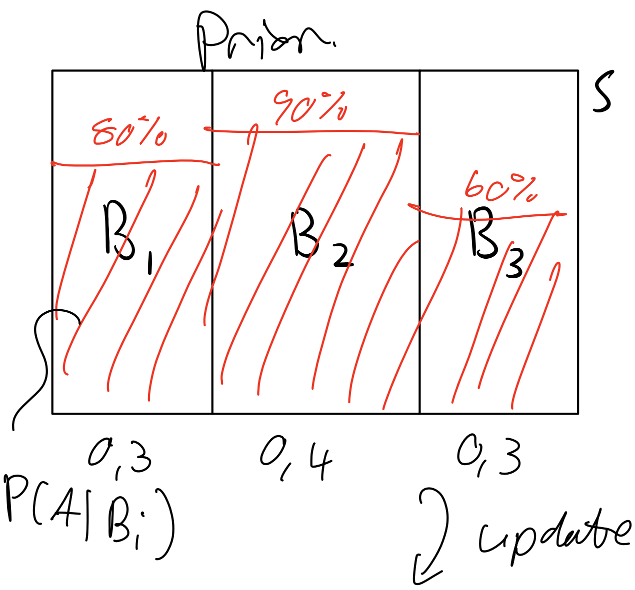
likelihood. Prior belief.

guilty evidence.

$$\boxed{P(B|A) = \frac{P(A|B)\cdot P(B)}{P(A)}}$$

Posterior. Probability evidence

Example 1.17



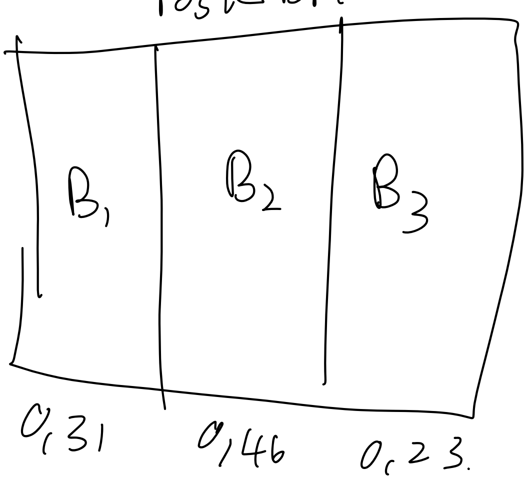
$$P(A) = P(A|B)\,P(B) + P(A|\bar{B})\,P(\bar{B})$$

$$P(B_3|A) = \frac{P(A|B_3)\cdot P(B_3)}{P(A)}$$

$$= \frac{0{,}6 \quad 0{,}3}{0{,}78}$$

$$= 0{,}23.$$

update.



$$P(B_2|A) = 0{,}46$$

$$P(B_1|A) = 0{,}31$$

Story time

You think your stereotype means:

~~$P(\text{farmer} \mid \text{description}) = 10\%$~~

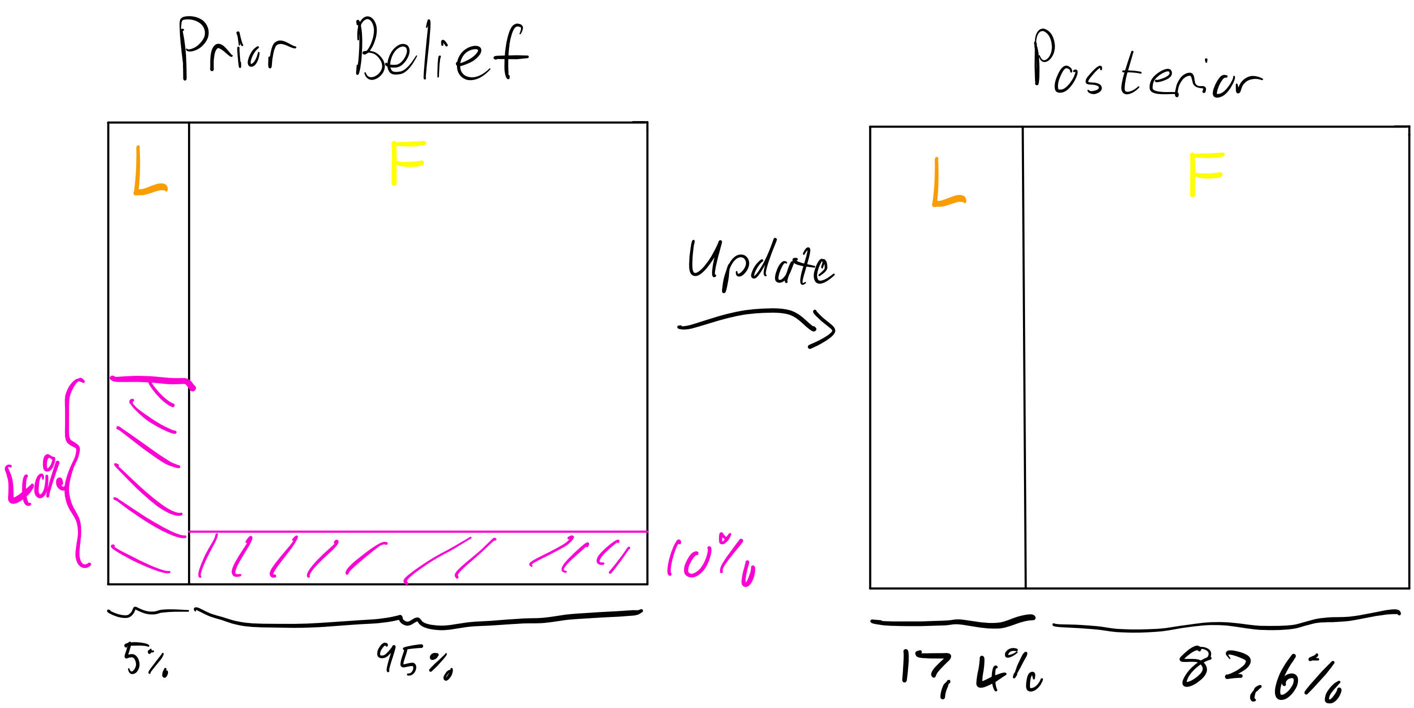
~~$P(\text{librarian} \mid \text{description}) = 40\%$~~

What it actually tells you:

$P(\text{description} \mid \text{farmer}) = 10\%$

$P(\text{description} \mid \text{librarian}) = 40\%$

What is ratio of farmers to librarians? What is your prior belief? 20 : 1



Bayesian Thinking

Prior.

Does God exist? $P(G) = 1\%$ $P(\bar{G}) = 99\%$

Evidence:
- Universe fine-tuned for life.
  cosmological const $P(\cdot) = \frac{1}{10^{120}} = 0{,}001$
- DNA 3,2 b base pairs.
  100 bp. $P(\cdot) = \frac{1}{10^{60}}$ 10^{40} chances. $P(\cdot) = \frac{1}{10^{20}}$
- Suffer exists.

$$P(G \mid \text{evidence}) = \frac{P(\text{evidence} \mid G)\, P(G)}{P(\text{evidence} \mid G)\, P(G) + P(\text{evi} \mid \bar{G})\, P(\bar{G})}$$

$$= \frac{0{,}9 \cdot 0{,}01}{0.9 \cdot 0{,}01 + (0{,}001) \cdot 0{,}99}$$

$$= 0{,}90$$