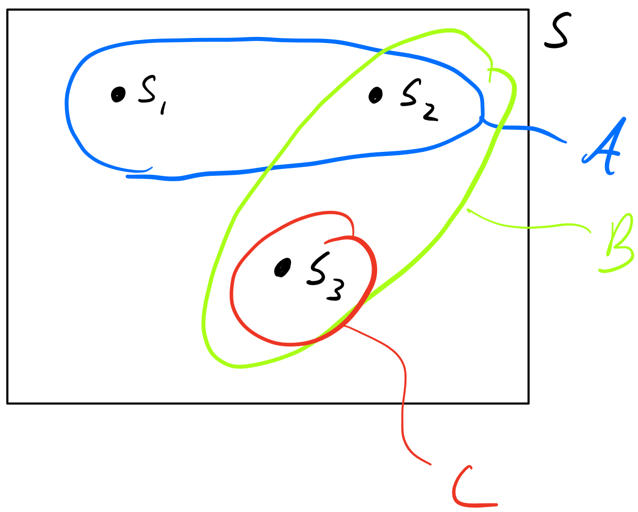
Recap



$$P(A) = \frac{2}{3}$$

$A \cap B \neq \emptyset$ not ME

$A \cap C = \emptyset$ is ME

$$P(A \cup C) = P(A) + P(C)$$

Axioms

Axiom 1: $P(A) \geq 0$ Non-negativity

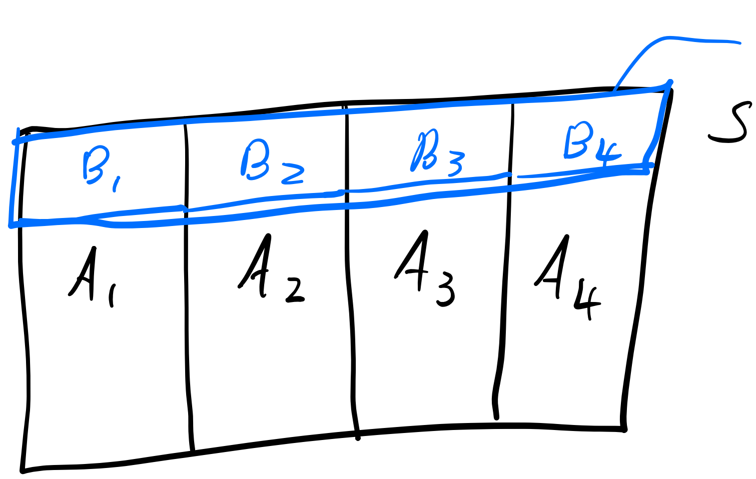
Axiom 2: $P(S) = 1$ Normilization

$\Big\} \; 0 \leq P(B) \leq 1$

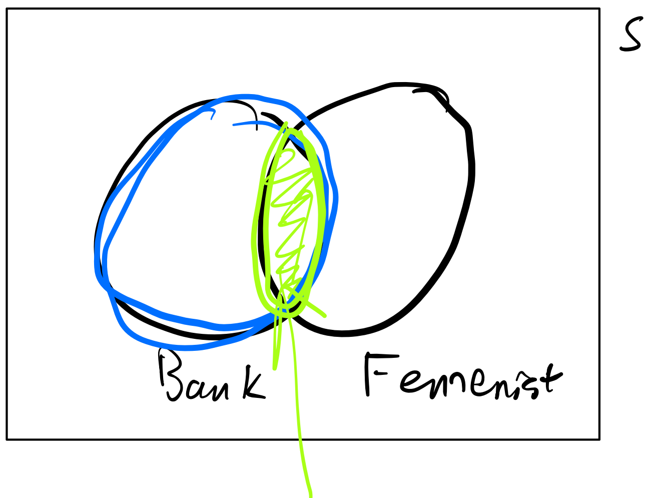
Axiom 3: $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$

$A_i \cap A_j = \emptyset \quad \forall \; i \neq j$ Additivity.

$$P(B) = \sum_{i=1}^{4} P(B_i)$$



Story



Proofs

- $P(\bar{A}) = 1 - P(A)$

$S = A \cup \bar{A}$ $\qquad \underline{P(S) = 1}$ Axiom 2

$P(\underline{A \cup \bar{A}}) = P(S) = 1$ Axiom 3

$P(A) + P(\bar{A}) = 1$

$P(\bar{A}) = 1 - P(A)$

• $P(\emptyset) = 0$

$A = \emptyset$

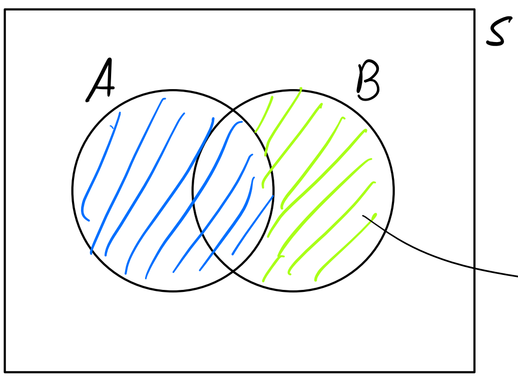
$\bar{A} = S$

$$
\begin{aligned}
P(\emptyset) = P(A) &= 1 - P(\bar{A}) \\
&= 1 - P(S) \quad \text{Axiom 2} \\
&= 1 - 1 \\
&= 0
\end{aligned}
$$

• $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

for $A \cap B \neq \emptyset$

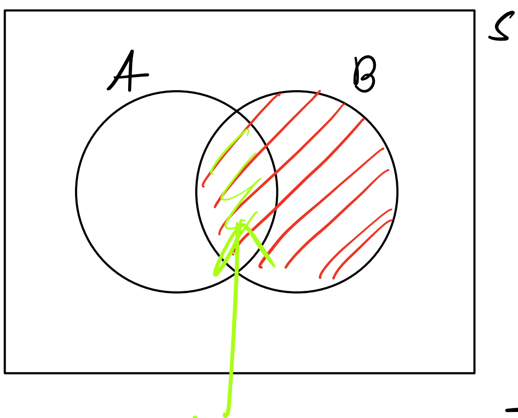
NOT ME



$A \cup B = A \cup (\bar{A} \cap B)$

$P(A \cup B) = P(A) + P(\bar{A} \cap B)$

$B = (A \cap B) \cup (\bar{A} \cap B)$



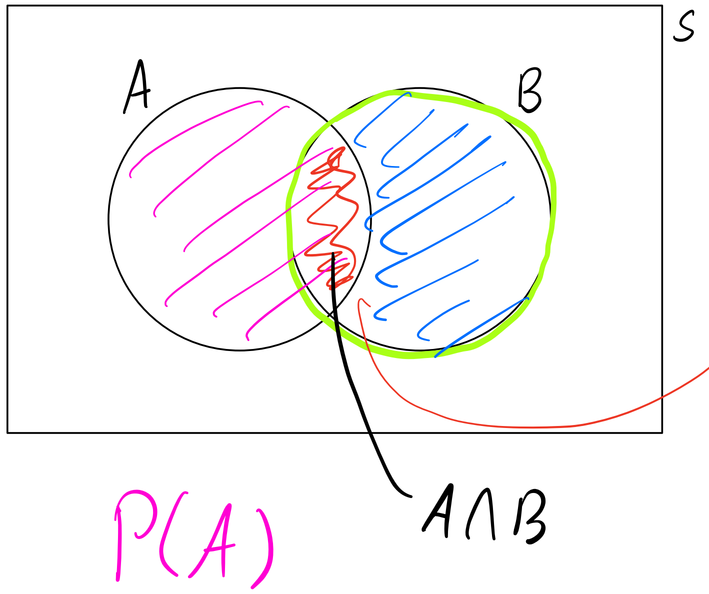
$A \cap B$

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

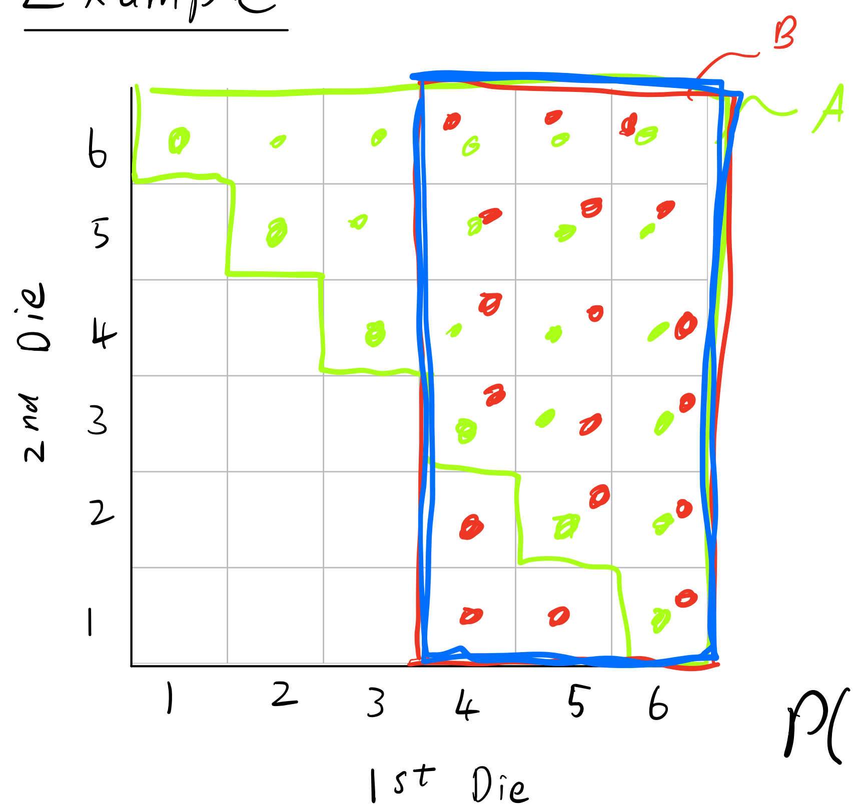
$$\Rightarrow \boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Conditional Probability



$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Example



$$P(A) = \frac{21}{36}$$

$$= 0{,}583$$

$$P(B) = \frac{18}{36} = \underline{0{,}5}$$

$$P(A \mid B) = \frac{15}{18} = 0{,}83$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{15}{36}}{0{,}5}$$

$$= 0{,}83$$